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Effect of an inhomogeneous interphase zone on the bulk modulus of a particulate composite containing spherical inclusions

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ABSTRACT

The effects of an inhomogeneous interphase zone on the effective bulk modulus of particulate composites containing large concentrations of inclusions are analyzed. The composite is modeled as a suspension of elastic homogeneous hollow spherical particles in a continuous elastic matrix. An interphase zone surrounding inclusions where the matrix material has elastic moduli with radial variation that asymptotically assume a constant value far away from particles is then modeled. The related elastic problem of a single inclusion in a finite matrix subjected to a spherically symmetric load is considered and a closed-form solution in terms of hypergeometric functions is determined. This analytical solution is then employed to derive an explicit expression for the effective bulk modulus. A detailed parametric analysis is finally performed to investigate the influence of the geometric characteristics and elastic properties of the graded interphase zone for composites containing voids or solid/hollow inclusions.

1. Introduction

In particulate composites inclusions are dispersed in a homogeneous matrix of different material in order to improve the performance of the single original material under specific loading conditions. In the framework of classical approaches to the mathematical modeling of the mechanical behavior of elastic composites (see e.g. Ref. [2]), these mixtures are treated as being macroscopically homogeneous with effective properties which depend on the elastic properties of all phases of the composite and on the shape, distribution and volume fraction of inclusions.

Predictions of such effective elastic moduli have mostly been derived assuming matrix and inclusions perfectly bonded across a well-defined interface at which the elastic moduli vary sharply from those of the inclusions to those of the matrix. However, in practice, due to manufacturing processes, a transition zone around the inclusions can form. The elastic properties of such a transition zone are usually a combination of those in the inclusions and in the matrix which can extend over a large area so affecting strongly the macroscopic behavior of the composite material. Nanoparticle reinforced materials, as well as syntactic foams containing hollow inclusions, are examples of particulate composites in which the thickness of the transition zone is comparable with the inclusion characteristic size (see e.g. Refs. [8,9,11,14]). In this case, the above assumption of sharp interface is not adequate and an interphase zone with suitable elastic moduli is required to be included in the constitutive model.

A number of studies have dealt with modeling the interphase zone in predicting the overall moduli of composite materials. The interphase zone is treated as a distinct phase from those of inclusions and matrix and assumed either homogeneous with constant elastic moduli (see e.g. Ref. [4]) or inhomogeneous with changing properties. With reference to particulate composites containing spherical inclusions, Lutz and Zimmerman [7] assumed that the interphase elastic moduli vary continuously throughout the entire region outside of the inclusion according to a power law. They employed the series solution so derived to predict the effective bulk modulus for small to accurate values of inclusion concentration. More recently, Sburlati and Cianci [13] derived a closed-form solution in terms of hypergeometric functions to the problem of hydrostatic loading of an infinite elastic matrix containing a hollow spherical inclusion, surrounded by an interphase zone with elastic properties varying in radial direction. The interface between inclusion and interphase is still distinct, while the interface between interphase and matrix is not sharply defined: namely, the elastic properties of the interphase vary according to a power law and asymptotically approach the matrix elastic properties. On the basis of standard energy considerations, this solution is employed

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also to derive the effective bulk modulus of a composite material containing a dilute dispersion of such inclusions with interphase (see e.g. Ref. [2]).

The objective of this paper is to analyze the effects of an inhomogeneous interphase zone on the effective bulk modulus of particulate composites containing large concentrations of inclusions and perform a detailed parametric analysis to better understand the effects of the interphase zone on the elastic properties of the composite.

To this end, using the solution method adopted in Sburlati and Cianci [13], in Section 2 the geometrical model of a single inclusion in a finite graded matrix subjected to a spherically symmetric load is presented and the related analytical solution determined in closed-form. Section 3 lays out the procedure for predicting the effective bulk modulus for finite volume fractions of inclusions. Some numerical examples are shown in Section 4 for solid inclusions and in Section 5 for hollow inclusions. Finally, Section 6 concludes the paper with a final discussion on the results presented.

2. Single inclusion problem under hydrostatic stress

The geometric model here considered is shown in Fig. 1. A hollow spherical inclusion of inner radius a and outer radius b is embedded in a matrix phase (straightforwardly, $a=0$ corresponds to the particular case of solid inclusion and $a=b$ to the case of void). The inclusion is made of a homogeneous isotropic material of Lamè moduli λ_i and μ_i. The inclusion is enclosed in a concentric shell of the matrix material having outer radius R. Following Lutz and Zimmerman [7] and Sburlati and Cianci [13], in order to model an interphase zone surrounding the inclusion, the matrix material is assumed isotropic but graded in radial direction with Lamè moduli λ and μ which vary according to the same following power law. Thus, with reference to a spherical coordinate system, we have

$$\lambda(r)=\lambda_m+(\lambda_{ip}-\lambda_m)\left(\frac{b}{r}\right)^{\beta}=\lambda_m+\bar{\lambda}\left(\frac{b}{r}\right)^{\beta},$$
$$\mu(r)=\mu_m+\left(\mu_{ip}-\mu_m\right)\left(\frac{b}{r}\right)^{\beta}=\mu_m+\bar{\mu}\left(\frac{b}{r}\right)^{\beta}, \tag{2.1}$$

where: $b\le r\le R$ measures the distance from the interface with the inclusion; λ_{ip} and μ_{ip} are the values of the Lamè moduli at the interface with the inclusion; λ_m and μ_m are the asymptotic values of the Lamè moduli far away from the inclusion; parameter $\beta>0$ controls the moduli variation in the matrix. $\lambda_{ip}\neq\lambda_i$ and $\mu_{ip}\neq\mu_i$ correspond to a sharp interface with the inclusion. A stiffer or more compliant interphase than the matrix can be modeled suitably

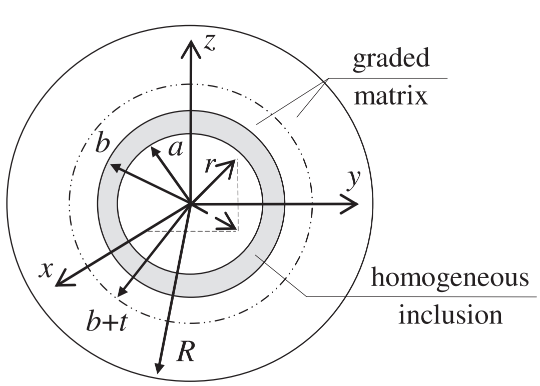

Fig. 1. Geometric model.

choosing, respectively, $\lambda_{ip}>\lambda_m$, $\mu_{ip}>\mu_m$ or $\lambda_{ip}<\lambda_m$, $\mu_{ip}<\mu_m$; for $\lambda_{ip}=\lambda_m$ and $\mu_{ip}=\mu_m$ the problem reduces to that of a single inclusion bonded to a homogeneous matrix. Moreover, larger values of β correspond to more localized interphases. On the other hand, being the interphase zone part of the matrix, no distinct interface with the matrix exists. As a consequence, the interphase thickness, say t, depends on β and cannot be regarded as an independent geometric parameter of the model. A criterion to relate t and β is discussed in Section 3. Finally, recalling that for isotropic materials Lamè moduli are related to bulk modulus as

$$k=\frac{1}{3}(2\mu+3\lambda), \tag{2.2}$$

from Eq. (2.1) it follows that also the bulk modulus of the graded matrix material varies according to the power law

$$k(r)=k_m+\left(k_{ip}-k_m\right)\left(\frac{b}{r}\right)^{\beta}=k_m+\bar{k}\left(\frac{b}{r}\right)^{\beta}, \tag{2.3}$$

where: k_{ip} is the value of the bulk modulus at the interface with the inclusion, whereas k_m is the asymptotic value far away from the inclusion.

Since it is of interest here to determine the effective bulk modulus, the single sphere model of Fig. 1 is considered subjected to a hydrostatic pressure p on its outer boundary ($r=R>b+t$). In this way, the boundary and continuity conditions are

$$\sigma_r^{(i)}(a)=0,\ \sigma_r^{(m)}(R)=-p$$
and
$$u^{(i)}(b)=u^{(m)}(b),\ \sigma_r^{(i)}(b)=\sigma_r^{(m)}(b), \tag{2.4}$$

where $\sigma_r^{(k)}$ and $u^{(k)}$ are the radial stress and displacement in the inclusion ($k=i$) or in the matrix ($k=m$).

Sburlati and Cianci [13] posed and solved analytically the mathematical problem of the single inclusion in an infinite graded matrix ($R\to\infty$). Their exact solution was expressed in terms of hypergeometric functions and explicit expressions for a hollow inclusion surrounded by an inhomogeneous interphase were derived and also specialized to the case of solid inclusion. On the basis of standard energy considerations, this explicit solution was then employed to derive a formula, approximated to the first order in inclusion volume fraction, of the effective bulk modulus for a composite containing a dispersion of such non-interacting inclusions which takes into account the effect of an interphase zone.

It is worthwhile to note that the same solution procedure can be adopted and suitably extended to solve the single sphere problem having finite R here under consideration. In this way, the radial displacement and stresses in the graded matrix assume the form

$$u^{(m)}(r)=\frac{A_1}{r^2}\Theta_1(r)+A_2 r\Theta_2(r), \tag{2.5}$$

and

$$\sigma_r^{(m)}(r)=\frac{A_1}{r^3}f_1(r)+A_2f_2(r)\ \text{and}\ \sigma_\theta^{(m)}(r)=\frac{A_1}{r^3}f_3(r)+A_2f_4(r), \tag{2.6}$$

where