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Systematic uncertainties in RF-based measurement of superconducting cavity quality factors

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ABSTRACT

Q_0 determinations based on RF power measurements are subject to at least three potentially large systematic effects that have not been previously appreciated. Instrumental factors that can systematically bias RF based measurements of Q_0 are quantified and steps that can be taken to improve the determination of Q_0 are discussed.

1. Introduction

The intrinsic quality factor, Q_0, of a superconducting cavity is an important measure of its performance. The ability to produce cavities with higher Q_0 could reduce capital and operating costs of future accelerators. Research into both the fundamental superconducting properties and preparation techniques required to achieving high quality factors is ongoing at many institutions [1–3]. To fully understand how cavity performance might be improved, systematic uncertainties in the measurements used to extract material properties must be well understood [4].

If the intrinsic coupling factor between cavity and coupler, β, is close to unity during testing Q_0 can be determined from direct measurement of RF losses in the cavity [6]. On the other hand, if the coupling is far greater than unity, cryogenic heat load measurements must be employed. Only RF measurement techniques will be considered here.

RF-based quality factor measurements commonly employ a circuit similar to that shown schematically in Fig. 1. The cavity is excited by a CW drive signal via a power antenna with coupling close to matched and the cavity field is monitored by a weakly coupled probe antenna. The forward, reflected, and transmitted powers: P_F, P_R, P_T are measured during steady state operation and the cavity decay time, τ, is measured when the power to the cavity is shut off.

The loaded quality factor, Q_L, can be determined from the angular frequency of the RF drive waveform, ω, and from the characteristic decay time, τ, of the stored energy when power to the cavity is shut off:

$$Q_L = \omega\tau. \quad (1)$$

Cavity quality factors (and hence the cavity decay time) in general depend on gradient. The decay time can be defined more precisely as the tangent of the decay curve at the beginning of the decay:

$$\tau = -\left(\frac{d\ \ln(U(t))}{dt}\right)^{-1}\Bigg|_{t_{Decay}=0}. \quad (2)$$

A common practice is to capture and fit the first 10% of the decay to calculate τ.

The cavity coupling can be determined by comparing the power dissipated in the cavity, P_D, to the reactive power, $P_X = \omega U = Q_L P_F$, when the cavity is precisely on resonance:

$$\beta = \frac{P_X}{P_D} = \frac{P_X}{P_F - P_R - P_T}. \quad (3)$$

The intrinsic quality factor can be determined from the cavity coupling and loaded quality factor as follows:

$$Q_0 = (1+\beta)Q_L \quad (4)$$

$$Q_{Ext} = (1+\beta^{-1})Q_L. \quad (5)$$

In practice a measured coupling, β^*, is determined by comparing the ratio on resonance of the reflected to forward power measurements:

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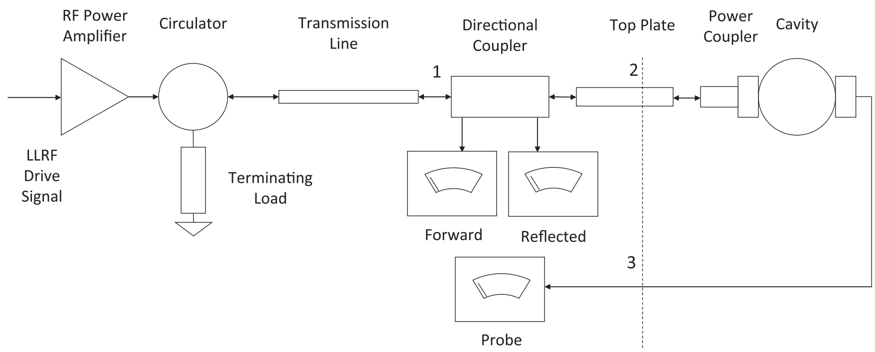

Fig. 1. Simplified schematic of the Fermilab Vertical Test Stand RF measurement system.

$$\beta^* = \frac{P_X}{P_F - P_R} = \left(\frac{1 + \sqrt{\frac{P_R}{P_F}}}{1 - \sqrt{\frac{P_R}{P_F}}} \right)^{\pm 1}. \quad (6)$$

The sign of the exponent in this equation is chosen to be positive (negative) if the cavity is over-coupled (under-coupled).

The coupling of the probe antenna is typically chosen to be much smaller (< 0.1) than the coupling of the power antenna. In this case $P_F - P_R \ll P_T$ and the difference between β and β^* is small:

$$\beta \approx \beta^* \left(1 + \frac{P_T}{P_F - P_R} \right) \quad (7)$$

RF power levels and cavity decay times can typically be determined with an accuracy of a few percent. If uncertainties in β and τ are independent the resulting uncertainty in Q_0 can be estimated using standard statistical methods for the propagation of uncertainties [5]:

$$\sigma_{Q_0}^2 = \left| \frac{\partial Q_0}{\partial \beta} \right|^2 \sigma_\beta^2 + \left| \frac{\partial Q_0}{\partial \tau} \right|^2 \sigma_\tau^2. \quad (8)$$

This leads to an uncertainty in Q_0 of:

$$\left\langle \left(\frac{\Delta Q_0}{Q_0} \right)^2 \right\rangle^{\frac{1}{2}} = \left(\left\langle \left(\frac{\Delta \tau}{\tau} \right)^2 \right\rangle + \frac{1}{\left(1 + \beta^{-1}\right)^2} \left\langle \left(\frac{\Delta \beta}{\beta} \right)^2 \right\rangle \right)^{1/2}. \quad (9)$$

Even under ideal conditions, quality factor measurements using this approach to are limited to accuracies of 5% or more [6,7].

Implicit in this approach, however, are three assumptions:

1. The forward and reflected waveforms are perfectly separated by the directional coupler during the coupling factor measurement.
2. No power is incident on the cavity during the decay time measurement.
3. The cavity is precisely on resonance during the coupling factor measurement.

Each of these three assumptions is violated in practice:

1. The imperfect directivity of the directional coupler used to separate the waveform incident on the cavity from the reflected waveform inevitably introduces some degree of cross-contamination between the signals.
2. Energy emitted into the reflected waveform from the cavity during the decay can re-reflect back from the circulator commonly used to isolate the RF power amplifier as energy incident on the cavity. The re-reflected energy may interfere constructively or destructively with the cavity field. This interference will systematically bias measured decay times.
3. Energy re-reflected from the circulator will also systematically shift the resonance frequency of the cavity-waveguide system from the true resonance of the cavity leading to systematic biases in the measured coupling factor.

Each of these three effects introduces additional uncertainties in Q_0 measurements that may be comparable to or larger than uncertainties associated with power meter calibration and decay time measurements. In the following, direct measurements, analytic calculations, and numerical simulations will be used to quantify uncertainties introduced by each of these effects. Steps that can be taken to reduce uncertainties from these sources will also be outlined.

2. Power meter calibration and decay time measurement uncertainties

Systematic uncertainties in Q_0 measurements from the calibration of the power meters used to monitor the cavity signals and from cavity decay time measurements have been discussed in detail elsewhere [6,7] but will be briefly outlined here for completeness.

If the fractional uncertainties in the calibration of each power meter (forward, reflected, and probe) are assumed to be the same, the uncertainty in the measured coupling factor is given by the following expression:

$$\left\langle \left(\frac{\Delta \beta^*}{\beta^*} \right)^2 \right\rangle_{PM}^{\frac{1}{2}} = \frac{\sqrt{2}}{4} \left| \beta^* - \beta^{*-1} \right| \left\langle \left(\frac{\Delta P}{P} \right)^2 \right\rangle^{\frac{1}{2}} \quad (10)$$

Fig. 2 shows the systematic uncertainty in the measured coupling factor as a function of coupling factor. The first-order analytic expression for the RMS uncertainty (green line) agrees well with Monte Carlo simulations (blue dots) over most of the range. The red line shows the peak uncertainty. As β^* becomes larger ($\beta^* \to 10$) the simulation results exceed the analytic estimates, indicating the analytic expression under-estimates the uncertainty for large values of β.

A previous analysis has estimated decay time measurement can be measured to an accuracy of 3% [7]. Additional systematic effects associated with energy reflected back into the forward wave by circulator impedance mismatches were not considered in that analysis will be discussed in detail below.

3. Directivity uncertainties

Dual directional couplers are commonly used to separate the voltage incident on the cavity from the voltage in the waveform