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Digital high-pass filter deconvolution by means of an infinite impulse response filter

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ABSTRACT

In the application of semiconductor detectors, the charge-sensitive amplifier is widely used in front-end electronics. The output signal is shaped by a typical exponential decay. Depending on the feedback network, this type of front-end electronics suffers from the ballistic deficit problem, or an increased rate of pulse pile-ups. Moreover, spectroscopy applications require a correction of the pulse-height, while a shortened pulse-width is desirable for high-throughput applications. For both objectives, digital deconvolution of the exponential decay is convenient. With a general method and the signals of our custom charge-sensitive amplifier for cadmium zinc telluride detectors, we show how the transfer function of an amplifier is adapted to an infinite impulse response (IIR) filter. This paper investigates different design methods for an IIR filter in the discrete-time domain and verifies the obtained filter coefficients with respect to the equivalent continuous-time frequency response. Finally, the exponential decay is shaped to a step-like output signal that is exploited by a forward-looking pulse processing.

1. Introduction

A gamma-ray detector system based on a semiconductor detector such as cadmium zinc telluride (CdZnTe, CZT) usually consists of the detector crystal, the analog readout electronics for the amplification of the detector signal, and a pulse processing unit. Nowadays, the pulse processing is mainly integrated by an application-specific integrated circuit or by a digital circuit in a field-programmable gate array (FPGA). As we recently showed [1], the front-end electronics can be appropriately implemented with a charge-sensitive amplifier with a continuous reset through an *RC* feedback circuit. This type of amplifier discharges the integrated detector current from the feedback capacitor *C* with the resistor *R*. Thus the typical signal shape with an exponential decay is seen at the output of the charge-sensitive amplifier. A well-known problem of the charge-sensitive amplifier with *RC* feedback is the ballistic deficit. This is caused by continuous discharge of the feedback capacitor, even though the current of the detector is integrated. If the ratio of the *RC* time constant over the integration time decreases, the ballistic deficit dominates the measured peak amplitude [1]. To eliminate this effect and to reconstruct the initial charge by a deconvolution of the exponential decay, Stein et al. presented the Moving Window Deconvolution (MWD) [2–6]. Later, Jordanov et al. [7–9] described the same approach [6]. However, both algorithms are derived by an analysis of the exponential decay in the time domain. As a result, a deconvolution of the exponential decay in the discrete-time domain is described by [6]

$$y[n] = x[n] + k \sum_{i=-\infty}^{n-1} x[i], \tag{1}$$

where $y[n]$ is the deconvolution of the signal $x[n]$, which is the value of the continuous signal $x(t)$ at the discrete time $t = nT$ with the sampling interval T. The value of k is set to $(1 - k')$, where $k' = e^{\frac{-T}{\tau}}$ is “the decay constant of the preamplifier transfer function for one sampling interval” [2]. The authors proposed an alternative value $k = \frac{T}{\tau}$ in [6] assuming $\tau \gg T$. Both parameters for Eq. (1) transform the exponential decay of the signal into a step-like signal, as shown in Fig. 1.

The discrete-time signal $x[n]$ shown in Fig. 1 corresponds to an output signal of an ideal charge-sensitive amplifier with a feedback resistor R and feedback capacitance C. For a rectangular shaped input current pulse with amplitude I, where the current flows in the time interval from t_a to t_b, the continuous-time output signal $x(t)$ of the amplifier is calculated by

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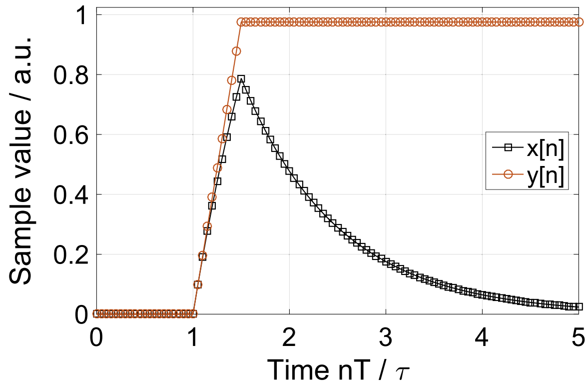

Fig. 1. A sampled output signal $x[n]$ of an amplifier with a typical exponential decay at sampling interval T (simulated). The decay has the time constant $\tau = 20\,T$, and the parameter k of the deconvolution is $k = 1 - e^{\frac{-T}{\tau}}$. This results in a step-like signal $y[n]$ with a corrected amplitude due to the ballistic deficit.

$$x(t) = IR\left[\left(1 - e^{\frac{t_a - t}{\tau}}\right)\theta(t - t_a) - \left(1 - e^{\frac{t_b - t}{\tau}}\right)\theta(t - t_b)\right]. \tag{2}$$

Here $\theta(t)$ is the Heaviside step function, and $t_b > t_a > 0$. Regarding Stein's approach for the MWD, the presented deconvolution is calculated by the recursive representation of Eq. (1), which is derived by

$$y[n] = y[n-1] + d \tag{3}$$

$$x[n] + k\sum_{i=-\infty}^{n-1} x[i] = x[n-1] + k\sum_{i=-\infty}^{n-2} x[i] + d \tag{4}$$

$$d = x[n] + (k-1)x[n-1] \tag{5}$$

$$y[n] = y[n-1] + x[n] + (k-1)x[n-1] \tag{6}$$

According to the time-shifting property of the z-transformation [11]

$$x\left[n - k\right] \overset{\mathcal{Z}}{\longleftrightarrow} z^{-k}X\left(z\right), \tag{7}$$

Eq. (6) can be rewritten as

$$\frac{Y(z)}{X(z)} = \frac{1 + (k-1)z^{-1}}{1 - z^{-1}}. \tag{8}$$

It is obvious that Eq. (8) is an equivalent of the generalized transfer function of an infinite impulse response (IIR) filter, which is defined as [11]

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{k=1}^{N} a_k z^{-k}} \tag{9}$$

For further clarification, the fundamental operation of the pulse shapers described by Stein et al. [2] or by Jordanov et al. [7] is a deconvolution of the transfer function of the amplifier and can be substituted by an IIR filter. Recently, Jordanov presented an unfolding-synthesis technique [10] that also demands an accurate deconvolution of the amplifier transfer function. Both constructed their digital algorithms intuitively by an extensive analysis of the signals in the time domain. By doing so, they neglected the established design methods for digital filters. Consequently, we will show a further analysis of the amplifier transfer function in the s-domain (frequency domain of continuous-time signals) and design the corresponding digital filter for the deconvolution in the z-domain (equivalent frequency domain of discrete-time signals). Finally, we will verify our algorithms with the signals of a CZT detector in conjunction with a charge-sensitive amplifier.

2. Discrete-time inverse amplifier transfer function

The charge-to-voltage transfer function of an ideal charge-sensitive amplifier with an RC feedback network and the voltage v_O at its output is given by [1]

$$H(s) = \frac{v_O}{Q} = \frac{sR}{1 + sRC}. \tag{10}$$

By normalizing the charge Q to the feedback capacitance C with $\frac{Q}{C} = v_Q$, the transfer function H_Q becomes

$$H_Q(s) = \frac{v_O}{v_Q} = \frac{s\tau}{1 + s\tau}, \tag{11}$$

where $\tau = RC$ is the characteristic time constant of the charge-sensitive amplifier. The transfer function is identical to that of a first-order high-pass filter. Therefore, the signal seen at the output of the amplifier is a convolution of the charge input signal and a high-pass filter. Moreover, as we want to reconstruct the input signal, the deconvolution of the high-pass filter is realized with the inverse transfer function of the amplifier. A deconvolution in the discrete-time domain requires an adequate approximation of H_Q^{-1} in the z-domain. Because the design methods for discrete-time filters that transform continuous-time filters are numerous, we will focus our investigations on a set of established methods and will test the accuracy of the transformation from the s-plane to the z-plane. At first, with the corresponding Laplace-transformation of the difference quotient of the continuous-time signal $x(t)$

$$sX \bullet\!\!-\!\!\circ \frac{dx(t)}{dt} = \lim_{h \to 0} \frac{x(t+h) - x(t)}{h}, \tag{12}$$

the equivalent difference quotient for the discrete-time signal $x(nT)$ is

$$\lim_{h \to T} \frac{x(nT+h) - x(nT)}{h} = \frac{x(nT+T) - x(nT)}{T} = \frac{x[n+1] - x[n]}{T} \tag{13}$$

By using Eqs. (7) and (13), the transformation of the s-domain to the z-domain is therefore derived by

$$s \longrightarrow \frac{z-1}{T}, \tag{14}$$

which is referred to as the forward difference method. In the same way, but by setting the difference quotient to $\frac{x(t) - x(t-h)}{h}$, the corresponding backward difference is defined by

$$s \longrightarrow \frac{z-1}{zT} \tag{15}$$

It is clear that these design methods replace the continuous-time differentials with a discrete-time difference. The exact relation of s and z in the context of the z-transformation is given by

$$z = e^{sT} \Longleftrightarrow s = \frac{1}{T}\ln(z), \tag{16}$$

which cannot be used for the expression of a discrete series of samples with respect to Eq. (7). Therefore, another substitution of s is derived by solving the differential equation corresponding to $H(s)$ by the approximation of an integral with the trapezoidal rule [11,12]. A replacement of s with