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The local GDQ method for the natural frequencies of doubly-curved shells with variable thickness: A general formulation

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ABSTRACT

The present paper aims to evaluate the natural frequencies of several doubly-curved shells with variable thickness. The general theoretical formulation allows to take into account various higher-order Equivalent Single Layer (ESL) theories in a unified manner, including the Murakami's function to capture the zig-zag effect. Such approach is able to study very well the dynamic behavior of a laminated composite shell, even in the presence of a soft-core. A general expression, which is able to combine different kinds of variations (such as linear, parabolic, exponential, sine-wave, Gaussian and elliptic shapes), is introduced to define the thickness profiles. In addition, the same formulation can be employed to localize such variations and to define, consequently, ribbed structures. Since the adopted structural model is two-dimensional, the shell reference surface represents the physical domain in which the governing equations are written. Thus, the differential geometry is necessary to define accurately the doubly-curved surfaces at issue. The fundamental system is solved numerically by means of a local approach of the well-known Generalized Differential Quadrature (GDQ) method. The matrices that allow to solve the problem in hand are banded, since only a part of the discrete grid points is considered. As a consequence, the computational effort is lower, if compared to the corresponding global version. The accuracy, reliability and stability of the present approach are proved by the comparison with the results available in the literature and the solutions obtained through three-dimensional FEM models.

1. Introduction

Shell structures are widely used in many engineering fields due to the curvature that characterizes their reference surface, which confers a peculiar stiffness towards various kinds of external stresses. In addition, shell design aims to spare materials and build light structures owing to their high strength-to-weight ratio. Car bodies, arteries, large roofs, space shuttles, boat hulls, fuselages, mechanical components, are all examples of shell structures present in automotive engineering, biomechanics, architecture, aerospace and naval industries [1,2]. Thus, it is really easy to daily come into contact with these structures. Nevertheless, their strength, related to the curvature effect, represents one of the most common obstacles that limits the use of shells due to the difficulties connected with their mechanical behavior analysis [3]. Therefore, several structural models have been introduced to overcome such issues. In fact, the huge number of papers available in the literature can easily prove that the study of doubly-curved shells is still an open topic. This aspect is the basis of the development of various higher-order theories that aim to capture the structural behavior in a more efficient manner [4,5]. The books by Qatu [6] and Reddy [7] drew attention to the advantages that structures made of composite materials can offer compared to traditional materials. Coupling these benefits with the structural features of shells just mentioned, remarkable improvements have been achieved in the mechanical behavior of such structures. Thus, the use of fiber-reinforced composites, and mainly of laminates, allowed a more rapid diffusion of laminated composite shells in many engineering fields, due to their enhanced structural efficiency. It should be recalled that composite structures are characterized by higher strength and stiffness, within a smaller thickness, with respect to the corresponding structural elements made of isotropic materials, for example. Therefore, nowadays composite shells are widely used

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in manufacturing of many aerospace and automotive components. On the other hand, the industrial manufacture is supported by a strong research work at academic level. Many recent articles are the proofs of the interest toward laminated composite structures [8–37]. For the sake of completeness, it should be remarked that in the last years the academic research aimed to develop new classes of material in order to limit the defects of laminated composite materials, such as residual stresses, spatial heterogeneity and delamination, and to enhance the structural behavior of composite structures. Thus, it is becoming easier to see several papers in the literature that deal with functionally graded materials [38–57], smart composites [58–60] and carbon nanotube-reinforced structures [61–66]. Another aspect that can affect the mechanical behavior of shell structures is their shape. In fact, an optimal distribution of material can defer shells collapse for buckling, alter bending behavior, and reduce vibration amplitudes. Variable thickness can be a way to perform a shape optimization process that aims to redistribute and increase the shell stiffness within the domain, with a lower structural weight if compared to the corresponding structure with constant thickness. Therefore, the main aim of the present paper is to examine the free vibration characteristics of laminated composite doubly-curved shells with variable thickness. As a brief and partial history of researches on this topic, it is worth to cite the work by Mizusawa [67], in which a complete analysis of isotropic plates with variable thickness is carried out. Afonso and Hinton [68,69] present the free vibration analysis of plate and shell structures with arbitrary thickness variation and support conditions by a Finite Element formulation. An alternative procedure based on the finite strip method is presented by Hinton et al. in Refs. [70,71] to deal with the same topic. The works by Kang and Leissa [72,73] investigate the free vibration frequencies and the mode shapes of thick spherical segment with variable thickness by means of a three-dimensional method. The differential quadrature rule is employed by Wu and Liu [74] to perform the free vibration analysis of circular plates with radially variable thickness and elastic restraints. Alternatively, an exact form solution is presented by Eisenberger and Jabareen [75] to deal with the same topic. An approach based on the three-dimensional equations of elasticity is followed by Leissa and Kang [76] to determine the natural frequencies of open paraboloidal shells of revolution with arbitrary thickness. The studies of Jiang and Redekop [77] are focused on the static and dynamic responses of linear elastic orthotropic toroidal shells of variable thickness with the aim of developing a solution based on the Sanders-Budiansky shell equations. Kang and Leissa present a three-dimensional method of analysis for determining the free vibration frequencies and mode shapes of complete paraboloidal shells of revolution with variable thickness [78]. In the work by Shufrin and Eisenberger [79] a semi-analytical procedure based on the Multi-Term Extended Kantorovich Method (MTEKM) is employed to evaluate the natural frequencies of several rectangular plates with a thickness profile described in polynomial form. A new finite element method is introduced by Liang et al. [80] to analyze the natural vibrations of circular orthotropic plates with variable thickness. The same dynamic problem related to circular cylinders is solved in a closed form by Duan and Koh [81]. The dynamic stiffness method is employed by Efraim and Eisenberger [82] to obtain the exact frequency analysis of thick spherical shell segments with variable thickness. The well-known Carrera's unified formulation is considered by Dozio and Carrera [83] for the free vibration analysis of quadrilateral plates by means of a new version of the Ritz-method. Due to several practical applications, the work by Kang [84] aims to evaluate the natural frequencies of joined thick conical-cylindrical shells of revolution with variable thickness. Laminated doubly-curved shells with variable thickness are considered by Amabili [85], who presents a higher-order theory for the large amplitude vibrations in curvilinear coordinates. The paper by Lal and Rani [86] is focused on the free axisymmetric vibrations of circular sandwich plates with a parabolic variation of thickness. Here, the first-order shear deformation theory is taken into account. A deep doubly-curved shell element is presented by Naghsh et al. [87] for the free vibrations of stiffened shells of revolution with variable thickness. The work by Bacciocchi et al. [88] investigates the natural frequencies of laminated composite doubly-curved shells, singly-curved shells and plates with a continuous thickness variation by means of the Generalized Differential Quadrature (GDQ) method. The results shown in their work can be considered as the starting point of the analysis presented in the present paper. Finally, for the sake of completeness, it should be recalled that it is possible to find many other works in the literature about variable thickness structures made of functionally graded materials [89–93]. Classically, the Finite Element Method (FEM) is a numerical tool that is mostly employed to solve many structural problems in their weak (or variational) form. On the other hand, the governing equations can be solved by means of numerical techniques that allow to obtain the solution considering the strong form of the same problems. One of them is the GDQ method [94], which in the present work is used to evaluate the derivatives of the dynamic fundamental system according to a local approach. A complete review about this technique, as well as other related methods, can be found in the review paper by Tornabene et al. [95], whereas some advanced applications are reported in the book by Wang [96]. Recently, the GDQ method is combined with the mapping technique, typical of FEM, to solve complex problems with mechanical and geometrical discontinuities by using a strong formulation. The accuracy and stability features of this approach are proven in the works [97–109]. In conclusion, the present paper is organized as follows. Firstly, a general higher-order shell theoretical model including variable thickness is presented. Secondly, the bases of the local version of the GDQ method are illustrated. Finally, the free vibrations of many laminated composites doubly-curved and ribbed shells are evaluated numerically.

2. Shell theoretical model

It is well-known that the three-dimensional theory of elasticity is the basis of many structural theories, which can be applied to the

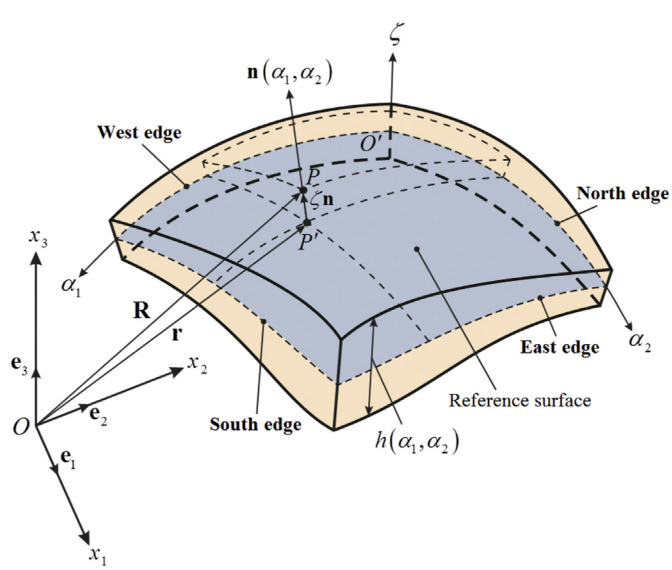

Fig. 1. Generic doubly-curved shell element with variable thickness.