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Efficient numerical modelling of the emittance evolution of beams with finite energy spread in plasma wakefield accelerators

T.J. Mehrling [a,*], R.E. Robson [b], J-H. Erbe [a], J. Osterhoff [a]

[a] Deutsches Elektronen-Synchrotron DESY, 22607 Hamburg, Germany
[b] Centre for Quantum Dynamics, School of Natural Sciences, Griffith University, Brisbane, Australia

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ABSTRACT

This paper introduces a semi-analytic numerical approach (SANA) for the rapid computation of the transverse emittance of beams with finite energy spread in plasma wakefield accelerators in the blowout regime. The SANA method is used to model the beam emittance evolution when injected into and extracted from realistic plasma profiles. Results are compared to particle-in-cell simulations, establishing the accuracy and efficiency of the procedure. In addition, it is demonstrated that the tapering of vacuum-to-plasma and plasma-to-vacuum transitions is a viable method for the mitigation of emittance growth of beams during their injection and extraction from and into plasma cells.

1. Introduction

Breakthroughs such as the demonstration of the energy doubling of 42 GeV electrons within a distance of less than a meter [1] established the credentials of plasma wakefield accelerators (PWFA) [2,3] as a viable future compact and affordable alternative to current conventional accelerators. In this plasma-based accelerator approach, a highly relativistic, high current *drive beam*, traverses a plasma target and thereby excites a large amplitude plasma wave. A *witness beam*, either externally injected or formed from plasma electrons by means of an internal injection method, is accelerated by virtue of the fields in the wake of the drive beam.

The increasing scientific interest in PWFAs eventually led to advances such as the experimental acceleration of distinct electron beams with high efficiency [4], and is primarily associated with the remarkable capability of these accelerators to generate and sustain extreme accelerating wakefields in excess of $\sim 10\,\mathrm{GV/m}$ during drive beam propagation in plasmas over meter-scale distances without substantial dephasing between drive and witness beam. This feature renders PWFAs an attractive technology candidate for the compact generation of brilliant X-rays or even for future compact and affordable particle colliders [5]. Besides the demands for high beam energies, these applications entail stringent requirements with regard to the beam quality. While a number of numerical studies [6–11] indicated the potential of PWFAs to generate high quality beams, one of the major challenges constitutes the preservation of these qualities during the witness beam extraction and transport to the interaction region. In addition, the beams may have to be transported in between successive plasma sections and re-injected without significant quality deterioration in staged acceleration concepts, such as proposed e.g. in [5].

As shown in previous works [12,13], the *transverse phase space emittance*, which is a figure of merit for the transverse beam quality, grows during the injection and propagation in a plasma target if beams are not matched appropriately. In addition, the phase space emittance of beams with finite energy spread and significant divergence may increase dramatically during the expansion in the vacuum downstream the plasma target [14,15]. The challenges to match beams into the plasma as well as to extract them can be substantially mitigated and the quality deterioration suppressed by the use of tapered vacuum-to-plasma and plasma-to-vacuum transitions, respectively, as indicated by several studies [13,16–19]. The rigorous investigation of the beam injection and extraction processes in PWFA typically involves the use of particle-in-cell (PIC) simulations. However, for the realisation of parameter scans e.g. for the optimisation of the beam parameters or of the plasma density profile, such an approach is inappropriately time consuming and computationally highly demanding.

This paper presents an alternative, computationally efficient semi-analytic numerical approach (SANA) for the investigation of the emittance evolution of beams with finite energy spread in PWFAs. Such an approach allows for the rapid optimisation of beam parameters and the longitudinal plasma profile in terms of the beam quality transport. This method was outlined in [20] and

* Corresponding author.
E-mail address: timon.mehrling@desy.de (T.J. Mehrling).

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is generalised in the present work to model beams with varying energy and energy spread. Section 2 reviews the physical basis and the moment procedure, and introduces the mathematical formulation of the semi-analytic numerical approach. The procedure is thereafter applied to a scenario in PWFA in Section 3. Results are compared to those obtained from PIC simulations with the 3D quasi-static code HiPACE [21] to establish the accuracy and efficiency of the procedure. In addition, this physical showcase study demonstrates the effectiveness of such realistic plasma profiles to mitigate the emittance growth during the injection and propagation in the plasma target and during the extraction. The paper is finalised with a summary and conclusion.

2. Mathematical model

2.1. Physical basis and moment procedure

The witness electron beam considered in the following propagates in positive z-direction in a plasma wakefield in the blowout regime excited by a drive beam. The witness beam particles are highly relativistic in axial direction and non- to mildly relativistic in the transverse direction i.e. $p_z \simeq \gamma \gg 1$, where γ is the Lorentz factor and p_z is the axial momentum normalised to $m_e c$. The beam particle phase space distribution function, which is in the following assumed to be symmetric in x and p_x with respect to zero, may therefore be characterised by $f = f(x, p_x, \zeta, \gamma; t)$. Here, t is the time normalised with respect to the inverse of a reference plasma density $\omega_{p,0}^{-1}$. The transverse particle offset coordinate x, the propagation axis coordinate z and the co-moving coordinate $\zeta = z - t$ are normalised to $k_{p,0}^{-1} = c/\omega_{p,0}$. The transverse momentum, denoted by p_x is given in units of $m_e c$. The temporal evolution of this particle distribution is prescribed by the *Vlasov equation* [22] which, in a manner similar to that described in Ref. [20], can be written to a good approximation as

$$(\partial_t + p_x/\gamma \partial_x + \partial_{p_x} F_x + \partial_\gamma F_z) f = 0 \tag{1}$$

where F_x and F_z are the transverse and longitudinal forces imposed onto the beam particles, normalised to $\omega_{p,0}\, m_e c$. The co-moving position hereby is assumed invariant, which is a good approximation for scenarios in PWFA where the slippage of witness beam electrons with respect to the speed of light frame over the regarded acceleration distance is typically negligible. Within the blowout of plasma waves, the transverse force is a function of the transverse coordinate and time $F_x = F_x(x, t)$ and the longitudinal force is a function of the co-moving position and time $F_z = F_z(\zeta, t)$ (compare e.g. [23]).

The aim is to compute the transverse phase space emittance

$$\epsilon = \sqrt{\langle x^2 \rangle \langle p_x^2 \rangle - \langle x p_x \rangle^2} \tag{2}$$

in an economical way. The averages $\langle \ldots \rangle$ are defined by

$$\langle \Phi(x, p_x, \zeta, \gamma) \rangle (t) = \frac{1}{N} \int dx \int dp_x \int d\zeta \int d\gamma\ \Phi f \tag{3}$$

in terms the particle number density f. The particle number is calculated via

$$N = \int dx \int dp_x \int d\zeta \int d\gamma, f. \tag{4}$$

While knowledge of f is essential for a complete picture of the electron bunch in phase space, an analytic solution of (1) is generally not possible. Finding numerical approximations for the distribution function f which satisfy the Vlasov equation e.g. by means of direct Vlasov solvers (see, e.g. [24–26]) or by means of PIC simulations (see, e.g. [27–29]) can be both computationally demanding and time-consuming.

However, if only *averages* such as in (2) are required then, as explained in [20], there is an alternative, computationally economical procedure, which yields these quantities *directly*, without the need to find the f first. Instead of solving for f in the Vlasov equation, one can multiply the Vlasov equation with Φ and then perform an integration by parts over the phase space variables, yielding the general moment equation [20]

$$\partial_t \langle \Phi \rangle = \langle p_x/\gamma \partial_x \Phi \rangle + \langle F_x\ \partial_{p_x} \Phi \rangle + \langle F_z\ \partial_\gamma \Phi \rangle. \tag{5}$$

As explained in [20] this *moment procedure* [30] generally yields an infinite chain of equations which, like for fluid models in configuration space, must be truncated via some Ansatz.

The transverse field within the blowout cavity depends linearly on the transverse offset to the propagation axis, $F_x(x, t) = -\hat{k}_x(t) x$. This allows us to express the equations for the phase space moments of interest, here $\Phi = \{x^2, x p_x, p_x^2, \gamma\}$, as

$$\partial_t \langle x^2 \rangle = 2 \langle x p_x / \gamma \rangle \tag{6}$$

$$\partial_t \langle x p_x \rangle = \langle p_x^2 / \gamma \rangle - \hat{k}_x \langle x^2 \rangle \tag{7}$$

$$\partial_t \langle p_x^2 \rangle = -2 \hat{k}_x \langle x p_x \rangle \tag{8}$$

$$\partial_t \langle \gamma \rangle = \langle F_z \rangle. \tag{9}$$

For *mono-energetic beams,* this set of equations is closed and may be related to the well known beam envelope equations [31]. However, the present paper goes beyond these envelope models, and investigates the transverse dynamics of realistic beams with non-zero, variable correlated or uncorrelated energy spread, and a variable energy.

2.2. Semi-analytical numerical approach

The following generalises the method introduced in [20] to solve the more comprehensive set of Eqs. (6)–(9). The basis of this method is the discretisation of the distribution function according to

$$f(x, p_x, \zeta, \gamma; t) \simeq \sum_{k=1}^{M} N_k \delta(\gamma - \gamma_k(t)) \delta(\zeta - \zeta_k) f_k(x, p_x; t) \tag{10}$$

where M is the number of mono-energetic subsets, N_k is the invariant number of electrons in one subset, $\gamma_k(t)$ is the time-dependent Lorentz factor of an energy subset and $f_k(x, p_x)$ is the normalised subset transverse phase space distribution function. It should be noted that γ here is an Eulerian phase-space quantity while $\gamma_k(t)$ is a Lagrangian quantity. The discretised distribution function (10) allows for the modelling of beams with correlated and uncorrelated energy spread. The total particle number is in this context given by

$$N = \sum_{k=1}^{M} N_k. \tag{11}$$

The co-moving position of a subset is hereby assumed invariable in compliance with the basic assumptions for the Vlasov equation (1).

The phase space quantities of interest for the study on the quality evolution of beams are separable $\Phi(x, p_x, \zeta, \gamma) = \Phi_t(x, p_x) \cdot \Phi_l(\zeta, \gamma)$, and the according phase space averages can thus be formed as follows:

$$\begin{aligned} \langle \Phi(x, p_x, \zeta, \gamma) \rangle &= \frac{1}{N} \int dx dp_x d\zeta d\gamma \Phi(x, p_x, \zeta, \gamma) f(x, p_x, \zeta, \gamma; t) \\ &= \frac{1}{N} \int dx\ dp_x d\zeta\ d\gamma \Phi_t(x, p_x) \Phi_l(\zeta, \gamma) f(x, p_x, \zeta, \gamma; t) \\ &= \frac{1}{N} \sum_{k=1}^{M} N_k \langle \Phi_l(\zeta, \gamma) \rangle_k \int dx\ dp_x\ \Phi_t(x, p_x) f_k(x, p_x; t) \\ &= \frac{1}{N} \sum_{k=1}^{M} N_k \langle \Phi_l(\zeta, \gamma) \rangle_k \cdot \langle \Phi_t(x, p_x) \rangle_k. \end{aligned} \tag{12}$$