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Electromagnetic self-consistent field initialization and fluid advance techniques for hybrid-kinetic PWFA code Architect

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ABSTRACT

The realization of Plasma Wakefield Acceleration experiments with high quality of the accelerated bunches requires an increasing number of numerical simulations to perform first-order assessments for the experimental design and online-analysis of the experimental results. Particle in Cell codes are the state-of-the-art tools to study the beam-plasma interaction mechanism, but due to their requirements in terms of number of cores and computational time makes them unsuitable for quick parametric scans. Considerable interest has been shown thus in methods which reduce the computational time needed for the simulation of plasma acceleration. Such methods include the use of hybrid kinetic-fluid models, which treat the relativistic bunches as in a PIC code and the background plasma electrons as a fluid. A technique to properly initialize the bunch electromagnetic fields in the time explicit hybrid kinetic-fluid code Architect is presented, as well the implementation of the Flux Corrected Transport scheme for the fluid equations integrated in the code.

1. Introduction

The electron Plasma Wakefield Acceleration (PWFA) technique [1] accelerates electrons through the electric fields induced in the wake of a high energy charged particle beam in a plasma. Proof of principle experiments [2,3] demonstrated to achieve accelerating gradients at least two orders of magnitude higher than the experiments with metallic accelerating cavities, limited by electric breakdown. Significant improvement in the PWFA process quality have been demonstrated accelerating an electron bunch (witness) in the wake of a higher charge electron bunch (driver) [3]. The overall witness bunch quality at the end of the plasma channel has been proven to be significantly dependent on the plasma and on the bunches parameters. To obtain plasma-accelerated bunches with quality suitable for applications, e.g. colliders or radiation sources, studies on feasible and robust working points for experiments beyond the accuracy degree of preliminary studies are necessary to aid the experimental efforts.

Three-dimensional Particle in Cell (PIC) [4] simulations are the state-of-the-art tool to catch the underlying physics of beam-plasma interaction. However, the massive computational requirements demanded by such simulations prevent their use for quick systematic scans for first experimental design assessment. For this reason considerable interest has been shown in methods which reduce the time required for plasma acceleration simulations. Examples are the use of GPUs [5] for PIC codes, the boosted-frame technique [6] or the quasi-static approximation (QSA) [7]. Reduced models as well have been proven to reduce the computational cost of simulations. Such models include the ones using the azimuthal Fourier decomposition [8]. An alternative approach to reduce the computational time is the hybrid kinetic fluid approach [9–11], which treats the beam as in a PIC code and the background plasma electrons as a fluid. A first example of this technique is the quasi-static code LCODE [9,10]. The hybrid approach drastically reduces the simulation time, since the background electrons are treated as a single fluid and not by a huge number of macroparticles.

In this paper we focus on the time-explicit hybrid kinetic-fluid serial code Architect [12], which treats electron bunches as in a PIC code and the background plasma electrons as a relativistic cold fluid. The code evolves the beam particles in the 3D3V space, while the fluid quantities and electromagnetic fields are integrated assuming cylindrical symmetry. A detailed comparison with a 3D PIC code, i.e. ALaDyn for PWFA [13,14], is under investigation. Preliminary results suggest good agreement for weakly non linear regimes, while for non linear regimes good agreement is found up to the region where the bubble closes up.

Architect has been developed to aid the studies for the PWFA experiments planned at SPARC_LAB facility [15]. The hybrid model allows to perform simulations of feasible SPARC_LAB working points in weakly nonlinear regimes [16] in less than one hour on a

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single cpu with no need of parallelization, making it suitable for systematic scans.

Quasi-static approximation is based on the assumption that the plasma background quantities depend mainly on the comoving variable $\xi = z - ct$ (with z the propagation direction). Such assumption is violated for example in presence of longitudinal steep transitions in the plasma channel initial density. Since Architect has also been conceived as a tool to perform systematic scans to aid the design of possible experimental set-ups of SPARC_LAB, time-explicit formulation was chosen for the code, to study also density profiles for which the QSA is not suited. The time explicit character of the code also allows to study the transition of the bunches from vacuum to plasma in presence of density profiles with strong variations in the longitudinal direction. At present several ramp lengths are being investigated, the shortest ramp length we are considering is less than 1 mm, the longest around 7 mm. A quick technique for the electromagnetic fields initialization in vacuum is presented, as well as the implementation of a high accuracy integration scheme for the fluid equations integrated in Architect.

2. Hybrid model for PWFA

Architect treats the relativistic electron beam as in a PIC code [4], the macroparticles move in the 6D phase space (3D space and 3D momentum space); the background plasma electrons are treated as a relativistic cold fluid. Plasma ions are treated as a uniform immobile background. The coupling between the two species is provided by the total electromagnetic fields, which are generated by the superposition of the two species: the sum of their current densities projected on the grid. The electromagnetic fields act as source terms for the beam particles equations of motion.

Denoting with $-\mathrm{e}$ the electron charge, $\mathbf{E}$ the electric field, $\mathbf{B}$ the magnetic field and c the speed of light, such equations are:

$$\dot{\mathbf{x}}_{\text{particle}} = \beta_{\text{particle}} c$$
$$\dot{\mathbf{p}}_{\text{particle}} = -\mathrm{e}\left(\mathbf{E} + c\beta_{\text{particle}} \times \mathbf{B}\right) \tag{1}$$

where for each particle of the kinetic beam $\mathbf{x}_{\text{particle}}$ the vector position, $\beta_{\text{particle}} c$ the vector velocity and $\mathbf{p}_{\text{particle}} = m_e \beta_{\text{particle}} c / \sqrt{1 - |\beta_{\text{particle}}|^2}$ the vector relativistic momentum (m_e is the electron mass) are defined. The dot derivative is intended as the total derivative. The electromagnetic fields act as source terms also for the background plasma electron fluid equations. Defining for the background the electron density n_e and momentum $\mathbf{p}_e$, they evolve according to the cold fluid equations: mass conservation equation and momentum conservation equation [17]:

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (\beta_e c n_e) = 0$$
$$\frac{\partial \mathbf{p}_e}{\partial t} + \beta_e c \cdot \nabla \mathbf{p}_e = -\mathrm{e}(\mathbf{E} + c\beta_e \times \mathbf{B})$$
$$\beta_e = \frac{\mathbf{p}_e}{m_e c \sqrt{1 + |\mathbf{p}_e / m_e c|^2}}. \tag{2}$$

The electromagnetic fields are integrated via Faraday's Law and Ampere–Maxwell's equation:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{E}$$
$$\frac{\partial \mathbf{E}}{\partial t} = c^2 \nabla \times \mathbf{B} + \mathrm{e}\mu_0 c^3 (n_e \beta_e + n_b \beta_b) \tag{3}$$

where $\beta_b c$ is the velocity for the electron bunch and n_b is the bunch density.

Architect treats the beam particles six-dimensionally: three components for the position and the momentum, which are integrated via Eq. (1); such a choice is used to avoid numerical issues on axis which arise when using particles moving in the $r-z$ space: the strategy to use particles moving in a 3D3V phase space avoids on axis numerical noise. The fluid and electromagnetic quantities evolve according to Eqs. (2) and (3), which are integrated with the assumption of cylindrical symmetry on a $r-z$ grid. The fluid Eqs. (2) are integrated through an operator splitting technique: first the fluid advection part is integrated through a Flux Corrected Transport (FCT) scheme [18,19]; then the fluid quantities are advanced using the electromagnetic fields as source term.

To save memory and simulation time the classical moving window technique is used, thus the electromagnetic and fluid quantities are integrated only in a window moving with the beam center of mass. Due to the cylindrical symmetry assumption, symmetric boundary conditions are implemented on the z axis and free flux boundary conditions are implemented on the other edges of the window (see Fig. 1).

The code loop for each timestep proceeds as follows: the beam particles current density $\mathbf{J}_b = n_b \beta_b c$ is projected on the grid using standard PIC techniques [4]. The fluid current density $\mathbf{J}_e = n_e \beta_e c$ is computed on the grid. The superposition of the two species current is then used as source term to integrate Eqs. (3) with a Finite Difference Time Domain scheme [4]. The updated electromagnetic fields are then used to advance the beam particles in the phase space with PIC techniques [4] and to advance the fluid quantities. The loop in detailed form, summarized in Fig. 2, is iterated.

3. Electromagnetic fields initialization

The self-consistency of the time-explicit approach demands a proper initialization of the electromagnetic fields at the first iteration. Assuming a plasma background initially at rest and an electron bunch in vacuum at the entrance of the plasma channel, the problem reduces to the proper computation of the bunch electromagnetic fields.

Since the typical energy spread of SPARC_LAB cases of interest is of the order of $\approx 0.1\%$, the bunch can be considered as static in a Lorentz reference system moving with its characteristic velocity β_0c in the propagation direction, which we assume to be z. In the laboratory frame the bunch potential Φ can be found Lorentz-transforming the Poisson equation in the laboratory frame [14]:

$$\left[\frac{1}{\gamma_0^2}\frac{\partial}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\right]\Phi = -q\frac{n_{b,0}}{\varepsilon_0} \tag{4}$$

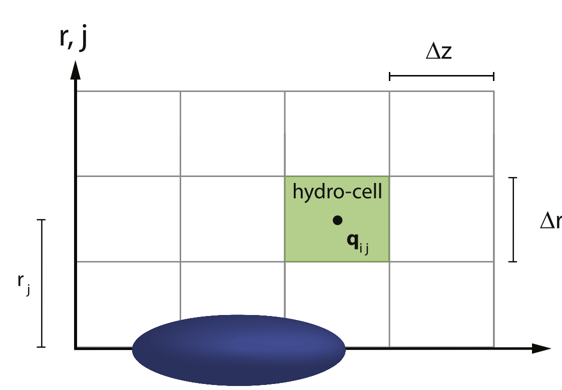

Fig. 1. Architect moving window around an electron bunch (in blue). In grey the grid on which the fluid equations are integrated. The hydro-cell in which the fluid quantities are cell-centered is highlighted in green. (For interpretation of the references to colour in this figure caption, the reader is referred to the web version of this paper.)