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Planar auxeticity from elliptic inclusions

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ABSTRACT

Composites with elliptic inclusions of long semi-axis a and short semi-axis b are studied by the Finite Element method. The centres of ellipses form a square lattice of the unit lattice constant. The neighbouring ellipses are perpendicular to each other and their axes are parallel to the lattice axes. The influence of geometry and material characteristics on the effective mechanical properties of these anisotropic composites is investigated for deformations applied along lattice axes. It is found that for anisotropic inclusions of low Young's modulus, when $a + b \to 1$ the effective Poisson's ratio tends to -1, while the effective Young's modulus takes very low values. In this case the structure performs the rotating rigid body mechanism. In the limit of large values of Young's modulus of inclusions, both effective Poisson's ratio and effective Young's modulus saturate to values which do not depend on Poisson's ratio of inclusions but depend on geometry of the composite and the matrix Poisson's ratio. For highly anisotropic inclusions of very large Young's modulus, the effective Poisson's ratio of the composite can be negative for *nonauxetic* both matrix and inclusions. This is a very simple example of an auxetic structure being not only entirely continuous, but with very high Young's modulus. A severe qualitative change in the composite behaviour is observed as a/b reaches the limit of 1, i.e. inclusions are isotropic. The observed changes in both Poisson's ratio and Young's modulus are complex functions of parameters defining the composite. The latter allows one to tailor a material of practically arbitrary elastic parameters.

1. Introduction

The negative Poisson's ratio (NPR) materials [1,2] coined *auxetics* [3], have attracted an increasing interest during last three decades. There are several mechanisms leading to the auxetic behaviour of solids [1,4,5,3,7–10] One of them is the rotating rigid units mechanism which has been a subject of intensive studies [11–15]. As it is well known, that mechanism acts on various length-scales: from nano to macro scale [11–16] and can be relatively easy applied to engineering of materials with negative Poisson's ratio [17–19]. Namely, one can cut holes of certain shapes in various (periodic) manners in order to obtain rigid units connected with narrow regions imitating the hinges allowing the rigid units to rotate [17,18,20–24]. The elliptic voids were introduced for the first time in the paper by Bertoldi and co-workers [25]. (In fact, the elliptic shape of the initially circular inclusions was a consequence of buckling accompanying the uniaxial compression.) Once the periodic pattern of holes is cut, the voids can be filled with a material of different mechanical properties. This leads to the idea of periodic composites made of auxetic and/or conventional materials.

The concept of composites with auxetic inclusions was investigated in a series of publications by Wei and Edwards [26–29] who proposed approximate analytic solutions for three-dimensional random composites of auxetic inclusions in frames of the mean field theory. Recently, the notion of auxetic phase as a constituent of a composite was discussed by Assidi and Ganghoffer [30] and Strek and co-workers [31–34]. An interesting auxetic composite material which becomes even more auxetic during stretching was studied by Hou et al. [35]. Christensen [36] considered effective properties of a composite consisting of elastic matrix with rigid spherical inclusions. The latter work is worth to be mentioned here because one of the aims of the present paper is the analysis of the limit of rigid inclusions. Discussing composites containing anisotropic inclusions, one should not forget about the important role of orientation of inclusions, discussed e.g. in the work by Banks-Sills and co-workers [37].

In the present work, the attention is focused on effective mechanical properties of systems with elliptic inclusions of Young's modulus varying from 0 (void inclusions) to ∞ (perfectly rigid

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inclusions). Axes of neighbouring ellipses of centres forming a square lattice (of axes x, y) are perpendicular to each other and parallel to x or y axis, respectively. The aim of this research is to establish the effective (macroscopic) mechanical properties of such a composite. The composite is not isotropic. However, for simplicity, this work is focused on mechanical properties along the x axis being the direction of applied force. (Studies of the general case will be the next step.) The analysis is performed by the finite element method, as it has already been successfully used in solving other composite structures [38,39]. We should add here that a preliminary stage of this research was discussed during the 8th Workshop on "Auxetics and Related Systems" on September 2011 in Szczecin, Poland [40].

The article is organized as follows. In the Section 2 the model is introduced, i.e. geometry of the composite and materials forming it are defined. In the Section 3 the method of solving the model is described. In the Section 4 the results are presented and discussed. Section 5 contains the summary and conclusions. Finally, in the Appendix A the convergence and accuracy of the numerical solutions are considered and the Appendix B contains results for a few geometries that were not included in the article body to avoid the legibility reduction.

2. The model

The subject of present studies are planar (2D) structures consisting of the matrix, wherein the elliptical inclusions are arranged in the periodic manner, see Fig. 1a. As shown in Fig. 1b, the centers of inclusions form the square lattice of the lattice constant $D = L_0/2$, where D is further taken as a unit on length and L_0 stands for the square-shaped periodicity box dimension. The elliptical inclusions have two possible orientations, horizontal and vertical, with nearest neighbours being always perpendicular to each other, see Fig. 1a. The unit cell of the analyzed structure is presented in Fig. 1b. The parameters A and B shown there are the only geometrical factors describing the semi-major and semi-minor axes of the ellipses respectively. In this paper dimensionless (divided by the lattice constant D) geometrical parameters $a = A/D$ and $b = B/D$ are also used.

The structure defined above can be seen as an analogue of either the rotating square model [11] with square-like elements of the matrix shown in Fig. 1c or as anti-tetrachiral arrangement of the latter elements [41,6,42]. It is convenient to define another (dependent) geometrical quantity $L_{gap} = (1 - a - b)D$ describing the thickness of the narrowest region of the matrix which is the smallest distance between the neighbouring inclusions, see Fig. 1a. The mentioned narrow region will be further referred to as a *neck*. These necks act as hinges in the rotating square model or as ribs in the anti-tetrachirals.

Apart from the geometrical parameters of the structure, the mechanical properties of the composite constituents are of essential importance. Elastic properties of isotropic materials within the linear elasticity theory are fully characterized by only two quantities [43]. In this work Young's modulus, E, and Poisson's ratio, ν, play this role. In the presented system the following parameters are variable: Poisson's ratio of the matrix (ν_{matr}) and inclusions (ν_{incl}), as well as the Young's modulus of the inclusions (E_{incl}). These parameters are as follows:

$$\nu_{matr} \in \{-0.99, -0.49, 0.0, 0.49\},$$

$$\nu_{incl} \in \{-0.99, -0.49, 0.0, 0.49\},$$

$$E_{incl} = 10^i (\text{in proper units}), \quad \text{where } i = -8, -7, \ldots, 6.$$

It is worth to add that the Young's modulus (E_{matr}) of the matrix plays the role of the reference value, being constant and equal to

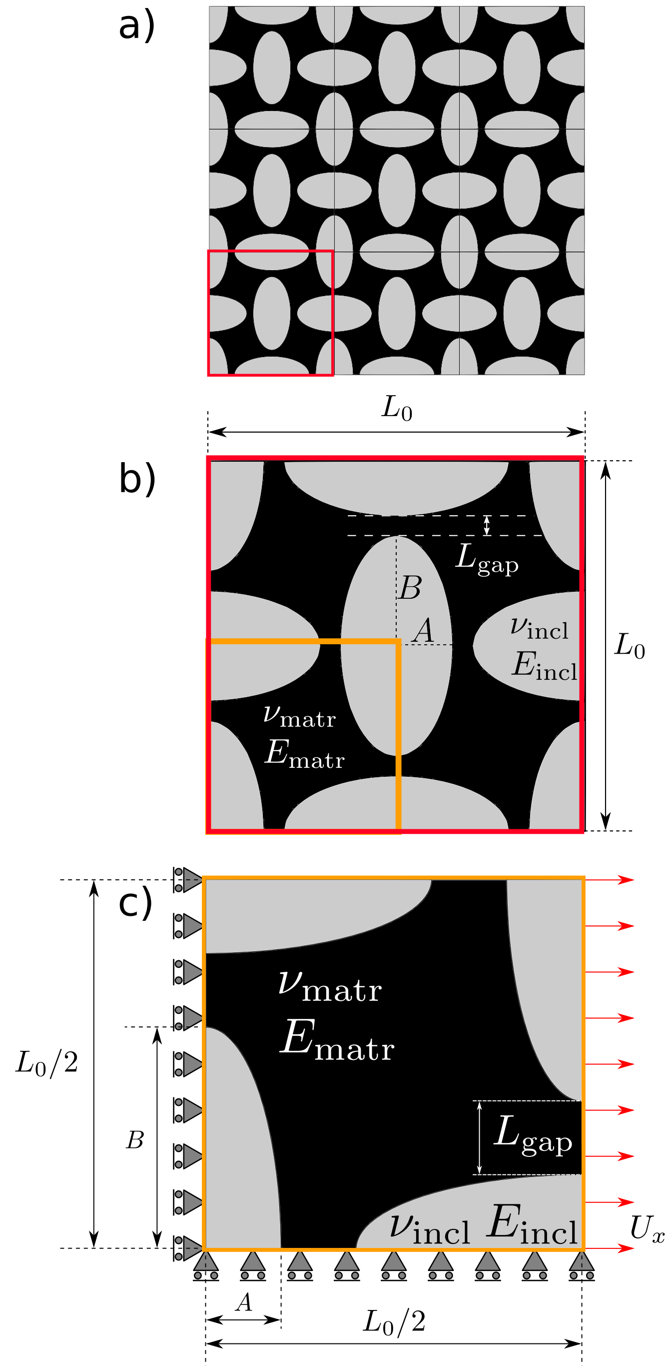

Fig. 1. Geometry of the structure (a), within a selected RUC (b) and simulated quarter with prescribed boundary conditions (c). All boundaries marked with orange lines in Fig. (c) remain straight lines and do not undergo rotations. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

unity (in proper units). Possibility of such a choice of the matrix Young's modulus is a consequence of the linear elasticity used in this work. Since the solution of the equations is scalable, it is enough to control the ratio E_{incl}/E_{matr}. Using a linear transformation the obtained solution can be then scaled to a particular value of E_{matr}. In turn, the values of the Poisson's ratio for both materials have been selected in the way presented above to avoid singularities which occur (in the 3D elasticity) for $\nu \in \{-1, 1/2\}$, whereas $\nu = -0.49$ is the negative approximation of the upper bound of Poisson's ratio (1/2) in 3D isotopic elasticity.