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Theoretical approach to predict transverse impact response of variable-stiffness curved composite plates

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ABSTRACT

This research studies the low velocity impact behaviour of variable stiffness curved composite plates. Since variable thickness within composite structures is recognised as an important factor on the performance of the structures, significant mathematical modelling to predict the impact response of these types of structure is essential. Varying thicknesses of sections is widely found in aerospace and automotive composite sub structures. It has been observed that changing of geometry of these sections can vary the dynamic response of anisotropic composite structures under a range of monolithic and dynamic loading conditions. Here we have used first order shear deformation theory to predict the contact force history of curved composite plates and the same approach was used for variable thickness composite plates, which provides the main novelty of this research. It was shown that the model developed here is capable of successfully predicting the response of variable stiffness composite plates with a range of layups and geometry designs under impact loading conditions.

1. Introduction

The use of composite materials in aerospace and automotive structures has greatly increased in the last decades as a result of their attractive structural properties. Beneficial properties such as low weight, high stiffness to weight ratio, high fatigue strengths and good corrosion and impact resistance are the most highlighted characteristics of these materials. It has been shown that failure modes of composite structures under low-velocity impact loading conditions are strongly dependent on the fibre type, resin type, lay-up, thickness, loading velocity and projectile type [1].

Many researchers have investigated different mathematical models to predict the impact behaviour of various composite materials and structures. Some of these models which are used to study the impact behaviour of composite structures by external objects are briefly discussed below.

Chai and Zhu [2] reviewed the numerical, mathematical and experimental methods used for the analysis of sandwich panels subjected to impact loading. They analysed the impact responses according to the main parameters, and consequently identified different classes of impact. The impact responses on sandwich structures were broadly categorised into two main groups, high-velocity and low-velocity impacts, with the focus on the low velocity impact. Khalili et al. [3] developed an analytical model to predict the impact force history which compared well with the experimental and analytical results in the literature. Their results showed that the stacking sequence of the face sheet has an insignificant effect on both the impact force and the contact duration. They also showed that if the case of zero in-plane forces is considered as a reference state, then positive in-plane forces increase the impact force and decrease the contact duration, while negative in-plane forces produce exactly the opposite effects, namely decreasing the impact force and increasing the contact duration. In another paper [4] they studied the dynamic response of a thin smart curved composite panel subjected to a low-velocity transverse impact. In their work shape memory alloys were used to reinforce the curved composite panel. A one-dimensional thermodynamic constitutive model by Liang and Rogers [ref] is used for estimating the structural recovery stress.

Shivakumar et al. [5] used a two-degrees-of-freedom model that consisted of four springs for bending, shear, membrane and contact rigidities to predict the impact response of a circular plate. In this model, the contact force and the contact duration for low-

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Nomenclature

E	Young's modulus (GPa)
$F(t)$	dynamic force
G_{12}	shear modulus (GPa)
h	thickness
t	beam thickness (mm)
R	radius
γ	Poisson's ratio
ρ	density
M_1	mass of plate
M_2	mass of striker
K1	stiffness constant of plate
K2	stiffness constant of striker
K_2^*	effective contact stiffness
V	impact velocity
u, v, w	displacements in x, y, z coordinate systems
x,θ,z	axial, circumferential and radial coordinates for a curved shell
k	Mindlin shear correction factor
β_x,β_y	bending slopes in the $x-z$ and $\theta-z$ planes
$N_x,N_\theta,N_{x\theta}$	in surface stress resultants in cylindrical polar system
$M_x,M_\theta,M_{x\theta}$	bending and twisting moments per unit length
q_x,q_θ	surface load components along the axial x-axis & circumferential θ axes
A_{ij},D_{ij}	extensional, bending stiffness of a laminated shell
Q_x,Q_θ	transverse shearing force per unit length

velocity impact on circular laminates was calculated. Gong and Lam [6] used a spring–mass model having two degrees-of-freedom in order to determine the history of the contact force produced during impact. They included structural damping also in their model. Gong et al. [7] studied the elastic response of orthotropic laminated cylindrical shells to low-velocity impact. A spring-mass model was developed to determine the contact force between the shell and the striker. An analytical function for the contact force was derived in terms of material properties and the mass of the shell and the striker, as well as for the impact velocity. Caprino et al. [8] used a single degree-of-freedom system to analyse drop weight impact tests on glass/polyester sandwich panels. Anderson [9] described an investigation using a single degree-of-freedom model for large mass impact on composite sandwich laminates. The stiffness parameters of the model were derived from the results of a three-dimensional quasi-static contact analysis of a rigid sphere indenting a multi-layered sandwich laminate.

Nanda and Kapuria [10] showed that the orthotropic ratio of the composite has a significant effect on the wavenumbers for tangential and mid-surface rotation modes. The wave propagation response predicted by the classical laminate theory (CLT) differs widely from the first-order shear deformation theory (FSDT) prediction, for thin and thick, and shallow and deeply curved beams at both low and high frequencies. Thus, the CLT should not be used for wave propagation analysis of even thin curved laminated beams. More recently, Kavousi Sisi et al. [11] presented a theoretical method for low-velocity impact analysis of composite laminated beams with arbitrary lay-ups and various boundary conditions subjected to asynchronous/repeated impacts of multiple masses. Their results showed that the time of impact plays an important role in determining contact forces, beam displacements, absorbed energies by the beam and normal and shear stresses by positive and negative superposition of induced waves. Dinh Duc [12,13] investigated an analytical method for calculating the nonlinear dynamic response of eccentrically stiffened functionally graded double curved shallow shells resting on elastic foundations and being subjected to axial compressive load and transverse load. The non-linear equations were solved by the Runge-Kutta and Bubnov-Galerkin methods and their results characterised the effects of material and geometrical properties, elastic foundation and imperfection on the dynamical response of reinforced FGM shallow shells. Li et al. [14] studied low-velocity impact responses and impact-induced damage evaluation problems for the stiffened composite laminated plates based on the progressive failure model and layerwise/solid elements method (LW/SE). Ghasemnejad et al. [15,16] studied the Charpy impact behaviour of single and multi-delaminated hybrid composite beam structures. The Charpy impact test was chosen to study the energy absorption capability of a delaminated composite beam. It was shown that the composite beams with the position of delamination closer to the impacted surface are able to absorb more energy in comparison with other delamination positions in hybrid and non-hybrid ones.

Despite of all these research contributions which have investigated a range of theoretical models in order to study the dynamic and transient responses of composite structures, the effect of geometric changes in terms of variable layups (stiffness) within a composite structure remains an area that still requires further investigations. This research reports the development of a new mathematical model and to employ this to predict the impact response of curved composite panels with variable stiffness determined by thickness. A spring-mass model was used to predict the contact force between a striker and a curved plate with variable stiffness during an impact event. The effect of various parameters including layups, impact velocity and geometric change were investigated in the research reported here.

2. Theoretical formulations

The solution to the dynamic problem is presented in the form of expansions of the loads, displacement, and rotation functions as double Fourier series [7]. Each expression is based on a function of position and a function of time. Love's equations of motion for a curved shell of dimensions a and b, radius R and thickness h under external loads [7] are expressed as (Fig. 1):

$$\frac{\partial N_x}{\partial x}+\frac{1}{R}\frac{\partial N_{x\theta}}{\partial\theta}+q_x(x,\theta,t)=\rho h\ddot{u} \tag{1}$$

$$\frac{\partial N_{x\theta}}{\partial x}+\frac{1}{R}\frac{\partial N_\theta}{\partial\theta}+\frac{Q_\theta}{R}+q_\theta(x,\theta,t)=\rho h\ddot{v} \tag{2}$$

$$\frac{\partial Q_x}{\partial x}+\frac{1}{R}\frac{\partial Q_\theta}{\partial\theta}-\frac{N_\theta}{R}+q_n(x,\theta,t)=\rho h\ddot{w} \tag{3}$$

$$\frac{\partial M_x}{\partial x}+\frac{1}{R}\frac{\partial M_{x\theta}}{\partial\theta}-Q_x=\frac{\rho h^3}{12}\ddot{\beta}_x \tag{4}$$

$$\frac{\partial M_{x\theta}}{\partial x}+\frac{1}{R}\frac{\partial M_\theta}{\partial\theta}-Q_\theta=\frac{\rho h^3}{12}\ddot{\beta}_\theta \tag{5}$$