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Effect of agglomeration on the natural frequencies of functionally graded carbon nanotube-reinforced laminated composite doubly-curved shells

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This paper aims at investigating the effect of Carbon Nanotube (CNT) agglomeration on the free vibrations of laminated composite doubly-curved shells and panels reinforced by CNTs. The great performances of doubly-curved structures are joined with the excellent mechanical properties of CNTs. Several laminations schemes and various CNT exponential distributions along the thickness of the structures are considered. Thus, it is evident that the shell dynamic behavior can be affected by many parameters which characterize the reinforcing phase. A widespread parametric study is performed in order to show the natural frequency variation. The general theoretical model for shell structures is based on the so-called Carrera Unified Formulation (CUF) which allows to consider several Higher-order Shear Deformations Theories (HSDTs). In addition, a complete characterization of the mechanical properties of CNTs is presented. The governing equations for the free vibration analysis are solved numerically by means of the well-known Generalized Differential Quadrature (GDQ) method due to its accuracy, stability and reliability features.

1. Introduction

The works of Iijima [1,2] allowed to understand the great potentialities of Carbon Nanotubes (CNTs) and aroused the interest of many scientists, whose researches aimed to find a convenient application which was able to take advantage of them. Due to their excellent mechanical and thermal properties, CNTs have been seen immediately as the ideal candidate to reinforce those composite materials which can be employed in many technological fields, such as aerospace and mechanical engineering [3]. Nevertheless, the characterization of the mechanical properties of CNTs is still an open topic, as the huge number of papers can prove [4–20]. Several approaches can be found in the literature to define the mechanical behavior of such composites, employed especially in many structural applications in order to improve the dynamic response or to give a superior attitude in some buckling problems. These are the reasons that have led to the insertion of one or more CNT reinforced layers in the lamination schemes of laminated composite structures.

The extended Rule of Mixture [21–23] represents the simpler manner to evaluate the Young moduli, the shear moduli and the Poisson's ratios of a CNT reinforced layer, which has orthotropic features. The reader can find some applications of this approach in the works by Alibeigloo and Liew [24] and Alibeigloo [25], in which the theory of elasticity is used to investigate respectively the thermal and the dynamic behavior of some CNT reinforced composite structures. The same micromechanical procedure for the evaluation of the mechanical properties of these kinds of composites is used by Zhang, Lei and Liew to characterize the engineering constants of the material in their recent works [26–29], where an advanced version of the well-known Ritz method is employed to solve numerically the free vibration analysis.

A completely different approach is proposed by Shi et al. [30] to investigate the agglomeration effect of CNTs through a two-parameter theoretical model. The basis of this model consists in considering that the spatial distribution of CNTs in the matrix is irregular and consequently some areas of the composite have a higher concentration of reinforcing particles. Then, the homogenization based on the Mori–Tanaka scheme for granular composite

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materials [31] is employed to evaluate the effective mechanical properties of the composite, which in this case has isotropic overall features. The current approach is taken into account in the papers [32–34], where some parametric studies are presented in order to investigate the effect of CNT agglomeration on the vibrational behavior of some simple structures. In particular, Hedayati and Sobhani Aragh have considered the influence of graded agglomerated CNTs on the free vibration of annular sectorial plates resting on Pasternak foundation [32], whereas the natural frequencies of CNT reinforced cylindrical shells are evaluated by Sobhani Aragh et al. [33,34]. The present formulation represents the starting point of the current manuscript, which aims to study the effect of agglomeration on the natural frequencies of functionally graded carbon nanotube-reinforced laminated composite doubly-curved shells. In this way, the excellent mechanical properties of CNTs are joined with the outstanding performance and efficiency of doubly-curved shells, whose advantageous characteristics have given them much space in several engineering fields, such as civil, mechanical and naval ones [35–37].

In addition, shell qualities are enhanced mostly by the use of composite materials, such as laminated composites [38–45] or functionally graded materials (FGMs). This last class of composites has grown strongly in the last years to face the problems related to the stress concentrations and mechanical discontinuities of classical laminated structures [46–54]. It should be noted that the gradual variation of constituent volume fraction along the thickness typical of FGMs is employed to define the distribution of CNT reinforcing phase. Another way to improve the mechanical properties of shells and plates is given by the recent technological advancements. In particular, the possibility to arrange the reinforcing fibers along curvilinear paths attracted the attention of many researchers, as proven in the works [55–61]. Therefore, it is evident that the study of doubly-curved structures still offers many topics to develop due to the increasing production of innovative materials [62,63].

On the other side, the addition of more and more complex constitutive laws needs frequently the use of Higher-order Shear Deformation Theories (HSDTs) to predict correctly the mechanical behavior of such structures. The turning point in the development of HSDTs based on the definition of enriched kinematic models can be found in the works by Carrera [64–66], where the main issues that have led to the expression of the so-called Carrera Unified Formulation (CUF) are presented. A complete overview of these themes can be found in the books by Tornabene and Fantuzzi [67] and by Tornabene et al. [68,69], in which a general view on the higher-order structural theories is presented together with many numerical applications of structural mechanics related especially to doubly-curved shells. It should be remembered that the mechanics of laminated shell can be still found in those books which are recognized as the fundamental contributions to the research in this field [70–72].

Finally, it is well-known that a numerical approach can be needed to solve accurately the governing equations of each structural problem, such as the free vibration analysis. Recently, excellent results in terms of accuracy, stability and reliability have been reached by using the Generalized Differential Quadrature (GDQ) method, introduced by Shu in the nineties [73] to improve the Differential Quadrature (DQ) method presented by Bellmann and Casti [74] twenty years earlier. The reader can find a complete historical background of both GDQ and DQ methods, as well as the main aspects of these techniques and a rich bibliography, in the review paper [75]. The works [76–84] demonstrated that the results, achieved numerically by the application of GDQ, are accurate and stable if compared to the ones available in literature or obtained by many finite element commercial codes. For the sake of completeness, it should be noticed that the authors have developed a new technique which is able to consider discontinuities or irregular domains through the so-called domain decomposition and to solve the strong form of governing equations inside each element mapped on the computational element [85–97].

In conclusion, the current manuscript is organized as explained in the following. Firstly, the general shell theoretical model based on CUF is presented. Secondly, a complete treatise for the characterization of CNT mechanical properties is developed. Finally, some numerical applications for the free vibration analysis are shown in order to underline the influence of CNTs on the dynamic problem under consideration.

2. Theoretical model

2.1. Shell fundamental equations

The present formulation can be used to study the mechanical behavior of moderately thick and thick shells, for which the following relation is valid

$$0.01 \leq \max\left(\frac{h}{L_{\min}}, \frac{h}{R_{\min}}\right) \leq 0.2 \tag{1}$$

where L_{min} and R_{min} represent the smallest dimension and the minimum radius of curvature of the shell, respectively. In general, any point within the structure is described by the position vector $\mathbf{R}(\alpha_1, \alpha_2, \zeta)$, as it can be easily noticed from Fig. 1, where a doubly-curved panel with variable thickness $h(\alpha_1,\alpha_2)$ is depicted. Considering the global reference system $Ox_1x_2x_3$, the position vector $\mathbf{R}(\alpha_1, \alpha_2, \zeta)$ can be written as

$$\mathbf{R}(\alpha_1, \alpha_2, \zeta) = \mathbf{r}(\alpha_1, \alpha_2) + \zeta\mathbf{n}(\alpha_1, \alpha_2) \tag{2}$$

in which $\mathbf{r}(\alpha_1,\alpha_2)$ defines the vector that describes the reference surface (or middle surface) of the shell and $\mathbf{n}(\alpha_1,\alpha_2)$ represents the outward unit normal vector. It is important to underline that α_1, α_2 identify the orthogonal curvilinear principal co-ordinates upon the middle surface, whereas the parameter ζ is taken along the shell thickness. The basics of differential geometry [67] allow to express the position vector of the reference surface through its three components along the three global axes $Ox_1x_2x_3$ as

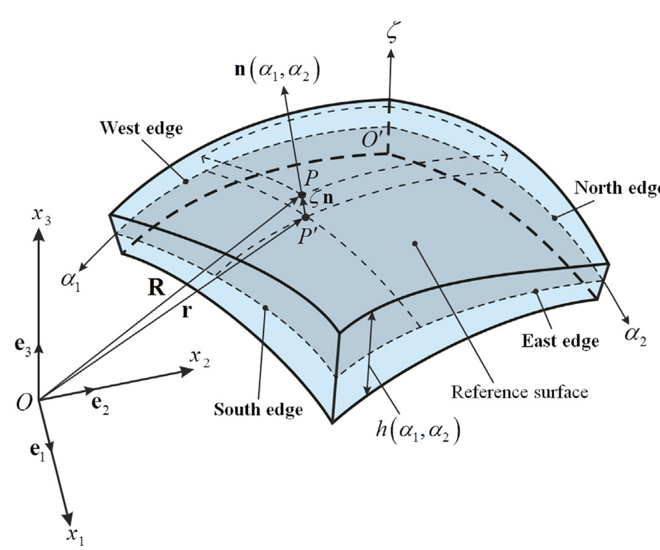

Fig. 1. Doubly-curved panel representation and description.