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Reissner's Mixed Variational Theorem and variable kinematics in the modelling of laminated composite and FGM doubly-curved shells

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ABSTRACT

The present article proposes a mixed displacements/transverse stresses approach for the free vibration analysis of laminated composite and FGM doubly-curved shells. The theoretical formulation is derived by combining Reissner's Mixed Variational Theorem (RMVT), Carrera's Unified Formulation (CUF) and the Ritz method. With the application of the RMVT the interlaminar equilibrium of the transverse normal and shear stresses is fulfilled a priori by exploiting the use of Lagrange multipliers. The transverse normal and shear stresses become primary variables within the formulation and are modelled with a Layer-Wise (LW) kinematics description. However, on the other hand, displacement variables, which describe the kinematics of the shell structures, are defined using Equivalent Single Layer (ESL), Zig-Zag (ZZ) and LW shell theories. The Mixed Hierarchical Trigonometric Ritz Formulation (MHTRF) is then used as solution technique to compute the natural frequencies of laminated composite and FGM doubly-curved shells. Several study-cases are addressed and the proposed RMVT-based shell models are assessed by comparison with both 3D elasticity and 3D Ritz solutions. The effect of significant parameters such as orthotropic ratio, stacking sequence, aspect ratio, lamination angle, length-to-thickness and radius-to-length ratios as well as volume fraction index on the dimensionless frequency parameters is discussed.

1. Introduction

Aerospace structures such as fuselage, wings, tales etc are generally made up of thin-walled cylindrical or spherical shell components. The use of the latter is justified because of the extraordinary load-carrying capability of shell structures. Furthermore, their structural efficiency is characterized by a high stiffness-to-weight and strength-to-weight ratios. Being nowadays aerospace applications extremely challenging then a deep and comprehensive investigation of the dynamic behaviour of cylindrical and spherical panels is required. In this respect, the majority of the shell theories which have been used in the last decades (see for example those provided in Refs. [1–20]) are inadequate to successfully address this task and the use of advanced shell models, like those developed in the present article, is needed.

With regards to the use of both approximation and analytical methods a considerable amount of research has been carried out and many articles have been written, some of them are briefly recalled below.

In particular, Leissa [21] provided a comprehensive free vibration analysis of doubly-curved shells with arbitrary in-plane shape. Qatu and Asadi [22] addressed the vibration analysis of doubly-curved shallow shells with arbitrary boundary conditions by using the Ritz method with algebraic polynomial displacement functions. Asadi et al. [23] employed a 3D and several shear deformation theories in order to carry out static and vibration analysis of thick deep laminated cylindrical shells. Ferreira et al. [24] used a wavelet collocation method for the analysis of laminated shells. The same author [25] combined a sinusoidal shear deformation theory with the radial basis functions collocation method to deal with static and vibration analyses of laminated composite shells. Recently, Fazzolari [26] developed and exact higher-order shell element by using the Dynamic Stiffness Method (DSM). Reviews on finite element shell formulations have been given by Denis and Palazzotto [27] and Di and Ramm [28].

Despite the high accuracy level obtained by using the shell models developed and presented in the articles touched upon hitherto, they still do not describe completely what have been referred to as C_z^0-requirements [29]. The latter consist in the

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capability of the developed shell model, to take into account the continuity of both displacements and transverse shear and normal stresses at the interface between two layers. Shell models based on the use of the Principle of Virtual Displacements (PVD) do not fulfil the C_z^0 -requirements because they do not account for the interlaminar continuity (IC) of the transverse stresses (see Fig. 1). To completely overcome this drawback, Reissner's Mixed Variational Theorem (RMVT) [30–39] has to be employed in the analysis of layered composite structures. By means of the RMVT the IC is fulfilled a priori by exploiting the use of the Lagrange multipliers which allow to variationally enforce the compatibility of the transverse shear and normal strains.

A further condition that must be satisfy in the modelling of composite structures is the Zig-Zag (ZZ) trend of the displacement components through the thickness direction. This behaviour is due to the intrinsic transverse anisotropy shown by composite structures.

Within the framework of the composite shell structures modelling the application of asymptotic methods must not be underrated and, in this respect, the articles of Fettahlioglu and Steel [40], Widera and Logan [41], Widera and Fan [42], Spencer et al. [43] and Cicala [44] worth to be highlighted.

Concerning the investigation of the dynamic characteristics of FGM doubly curved shells several articles have been proposed recently. Amongst these, Loy et al. [45] and Pradhan et al. [46] studied the free vibration behaviour of functionally graded cylindrical shells using Love's shell theory [6,5] and the Ritz method. The natural frequencies of simply supported functionally graded shallow shells were investigated by Matsunaga [47] using 2D higher order theory. A C^0 finite element formulation based on a HSDT was presented by Pradyumna and Bandyopadhyay [48] to cope with free vibration analysis of functionally graded curved panels. Zhao et al. [49] analysed the free vibration of functionally graded shells using the element-free Kp-Ritz method. Yang and Shen [50] investigated the free vibration and parametric resonance of shear deformable functionally graded cylindrical panels. Free vibration characteristics of functionally graded elliptical cylindrical shells were analysed by Patel et al. [51] using Finite Element Method (FEM) based on the theory with higher-order through-the-thickness approximations of both in-plane and transverse displacements. Vel and Batra [53,54,52] coped with several mechanical, thermal and thermo-mechanical problems of FGM structures. Closed-form solutions of free vibration problems of simply-supported multilayered shells made of functionally graded materials was presented by Cinefra et al. [55] where variable-kinematics shell models based on Carrera's Unified Formulation (CUF) were employed in conjunction with Reissner's Mixed Variation Theorem (RMVT). Free vibration analysis of functionally graded material cylindrical shells with holes was studied by Zhi-yuan and Huaning [56]. A three-dimensional vibration analysis of fluid-filled orthotropic FGM cylindrical shells was carried out by Chen et al. [57]. Sofiyev [58] studied the vibration and stability behaviour of freely supported FGM conical shells subjected to external pressure. Fazzolari [59] et al. studied the free vibration behaviour of FGM doubly-curved sandwich shells by using advanced hierarchical shell models. Many other articles based on the free vibration analysis of FGM shell structures which account for thermal effect, in-plane loadings, elastic foundations and the use of several boundary conditions can be found in Refs. [61–66,60]. An exhaustive review on the recent research on meshless methods for functionally graded shells has been presented by Liew et al. [67].

The present paper provides some advances in the use of the RMVT for the free vibration analysis of laminated composite and FGM doubly-curved shells. The mathematical formulation is essentially based on the combination of RMVT, CUF [68–71] and Ritz Method [72–79]. An accurate evaluation of the effectiveness of the higher order terms is carried out. Results are presented in terms of natural frequencies, the effect of stacking sequence, length-to-thickness ratio, radius-to-length and radius-to-thickness ratios as well as volume fraction index on the natural frequencies is discussed. Donnell-Mushtari's shallow shell-type equations are given as a particular condition. Finally some conclusions are drawn from the findings of the research.

2. Laminated composite shell geometries

The shell geometry and the parameters used to describe it, are shown in Fig. 2. The generic laminated shell is composed of N_l layers. Subscripts and superscripts k refers to the layer number which starts from the bottom of the shell. The layer geometry is denoted by the same symbols as those used for the whole multilayered shell and vice-versa. With α_k and β_k the curvilinear orthogonal coordinates (coinciding with the lines of principal curvature) on the layer reference surface Ω_k (middle surface of the k layer) are indicated. The z_k denotes the rectilinear coordinate in the normal direction with respect to the layer middle surface Ω_k. The Γ_k is the Ω_k boundary: Γ_k^g and Γ_k^m are those parts of Γ_k on which the geometrical and mechanical boundary conditions are imposed, respectively. These boundaries are herein considered parallel to α_k or β_k. For convenience the dimensionless thickness coordinate $\zeta_k = \frac{2\,z_k}{h_k}$ is introduced, where h_k denotes the thickness in the A_k domain ($A_k \in [z_k, z_{k+1}]$) is introduced. Most notably, in Fig. 1, $\boldsymbol{r}(\alpha,\beta)$ indicates the position vector of a point on the middle surface Ω of the shell, $\boldsymbol{R}(\alpha,\beta,z)$ is the position vector of a generic point within the volume occupied by the shell. At each point P of the middle surface $\boldsymbol{n}(\alpha,\beta)$ indicates the unit normal vector. In the shell geometry depicted in Fig. 2, the square of line elements, even known as the first fundamental form of the surface Ω_k is given as

$$ds_k^2 = \left(H_\alpha^k\right)^2 d\alpha_k^2 + \left(H_\beta^k\right)^2 d\beta_k^2 + \left(H_z^k\right)^2 dz_k^2 \quad (1)$$

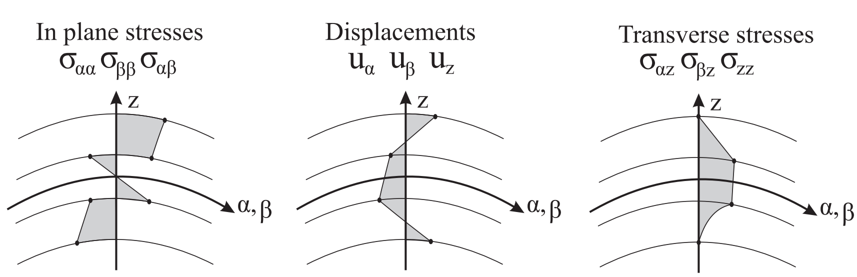

Fig. 1. C_z^0-requirements: interlaminar continuity condition of both displacements and transverse shear and normal stresses.