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Determinations of *in-situ* energy loss factors of point-connected composite plates

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ABSTRACT

Statistical energy analysis is a widely used high frequency vibro-acoustic analysis tool. It is based on power flow balance between subsystems. Its accuracy mainly depends on precise determination of energy loss factors. In the paper, three different types of composite structures (point-connected I, L and T types) made of laminated plates are of concern to determine *in-situ* energy loss factors by using power injection method[1] (PIM). The results are compared with those obtained by approximate analytical determinations, thus accuracy of numerical and experimental PIM is discussed for point-connected composite plates. Analytical loss factors are computed by an iterative procedure to consider indirect coupling for the structures having more than two subsystems.

1. Introduction

Statistical Energy Analysis (SEA) developed by R. H. Lyon [1] is one of the most powerful method in vibro-acoustic analysis of structural acoustic systems having uncertainty subjected to high frequency excitations at which deterministic techniques such as finite element and boundary element methods (FEM, BEM) using Monte Carlo simulation are inefficient due to theoretical restrictions and computational expenses [2,3]. SEA divides any complex system to its subsystems exhibiting common modal behavior. This provides very small system matrix thus makes the requirements of CPU time and memory usage meaningless. SEA sets power balance between these subsystems and each subsystem transmits power with proportional to coupling loss factor (CLF) and dissipates energy with proportional to damping loss factor (DLF). The energy is the primary variable of SEA method. Other dynamic variables such as displacement, velocity, pressure, etc., are found from the energy of vibrations. SEA predicts time-, spatial-, frequency-averaged responses, thus looses local information. Besides, it has several assumptions such as weak-coupling, proportional damping etc. However, it is still most widely used method for high frequency analysis.

In fact, the success of SEA mainly depends on the accurate prediction of energy loss factors. For simple structures, CLFs can be analytically determined using finite/semi-infinite system impedances/mobilities [1,4]; however, for relatively complex systems it requires auxiliary techniques based on numerical and/or experimental procedures. For this purpose, power injection method (PIM) [5–7] is one of the best alternatives for determining loss factors of SEA without separating structure to its subsystems, i.e. as *in-situ*. However, due to matrix inversion there may be singularities at some discrete frequencies. There are also different methods to determine coupling loss factors such as input power modulation technique [8], dual formulation [9], spectral element method [10], matrix fitting method [11]. Beside this, utilization of modal data is also used for CLF predictions; Seçgin [12] has developed a modal-based approach for the determination of SEA parameters including CLFs for directly connected composite plates having different orientation angles. Totaro et al. [13] has used a FEM-based modal approach for uncoupled subsystems to evaluate CLFs. Steel et al. [14] has also applied finite element model to determine CLF in a different manner. Fredö [15] has combined FEM and an SEA-like approach to determine power transmission between two plates in terms of energy flow coefficients.

In the present study, three different types of composite structures composed of laminated plates are considered. The plates are

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connected by bolts from three points to have I, L and T types of shapes. These shapes diversify the mechanism of energy transmission from one plate to another. Power injection method is then applied numerically and experimentally to predict *in-situ* energy loss factors, i.e, CLFs and DLFs between plates. For numerical implementation of the procedure, finite element method is adopted. Numerical and experimental predictions of *in-situ* energy loss results are evaluated by approximate analytical determinations. For the indirect coupling of structures having more than two subsystems, analytical computations are obtained by performing an iteration procedure. Therefore, the accuracy of numerical and experimental power injection procedures is clearly presented for these kinds of structures.

2. Mathematical considerations

2.1. Statistical energy analysis

SEA provides a power balance between subsystems assuming to consist of same type of resonant modes. Average power flow between coupled subsystems is proportional to the difference in the average modal energies. For S connected subsystems, the power balance equation for the ith subsystem is given as [1]:

$$2\pi f \eta_{ii}\langle \overline{E}_i\rangle + \sum_{j=1,\neq i}^{S} 2\pi f\left(\eta_{ij}\langle \overline{E}_i\rangle - \eta_{ji}\langle \overline{E}_j\rangle\right) = \langle \overline{P}_{in,i}\rangle \tag{1}$$

where f is the frequency in Hz, E_i is the subsystem total average energy, the $\langle\rangle$ denotes time averaging and over bar indicates frequency averaging,P_{in} is the input power, η_{ii} are DLFs, and η_{ij} are CLFs for the directly connected subsystems i and j, noting that $\eta_{ij} \neq \eta_{ji}$. If SEA equations are written for a two subsystems, Eq. (1) leads following matrix equation:

$$\begin{bmatrix} \eta_{11}+\eta_{12} & -\eta_{21} \\ -\eta_{12} & \eta_{22}+\eta_{21} \end{bmatrix} \begin{Bmatrix} \left\langle \overline{E_1} \right\rangle \\ \left\langle \overline{E_2} \right\rangle \end{Bmatrix} = \frac{1}{2\pi f} \begin{Bmatrix} \left\langle \overline{P_{in,1}} \right\rangle \\ \left\langle \overline{P_{in,2}} \right\rangle \end{Bmatrix} \tag{2}$$

For simple structures, CLF can be expressed as [1]:

$$\overline{\eta}_{ij} = \frac{\overline{\delta f_i}}{\pi f}\cdot \beta_{corr}\cdot \frac{\tau_{ij}}{2-\tau_{ij}}, \tag{3}$$

where,

$$\beta_{corr} = \frac{1}{\left(1+\left(\frac{1}{2\pi\left(\beta_{i,net}+\beta_{j,net}\right)}\right)^{8}\right)^{1/4}}. \tag{4}$$

Here, τ_{ij} is transmission coefficient and $\overline{\delta f_i}$ is Average Modal Spacing (AMS) and $\beta_{i,net}$ is net modal factor. Modal factor can be calculated by $f\eta_{i,net}/\overline{\delta f_i}$ in terms of net loss factor of subsystem i. Iteration is used for correct determination of coupling loss factor. Firstly, DLF is used in modal factor for calculating CLF. Then, net loss factor is used for satisfactory results. For plates, AMS is calculated by $\overline{\delta f_i} = 2\kappa c'_L/A$ in terms of the radius of gyration of the bending ($\kappa = h/\sqrt{12}$ for a plate of uniform thickness h, area A, and longitudinal wave speed $c'_L \equiv [E/\rho(1-\mu^2)]^{1/2}$, where E is Young modulus, ρ is density and μ is Poisson's ratio). For composite structures longitudinal wave speed is calculated as:

$$c'_L = \sqrt{c'_{Lx}\cdot c'_{Ly}} \tag{5}$$

Here, c'_{Lx} and c'_{Ly} represent longitudinal wave speed on x-direction and y-direction. For high frequencies, transmission coefficient of point connected structures is determined by Ref. [1],

$$\tau_{ij}(0) = \frac{4\cdot R_{i\infty}\cdot R_{j\infty}}{\left|\sum_{k=1}^{m} Z_{k\infty}\right|^2}. \tag{6}$$

Here, $Z_{k\infty}$ are impedances of subsystems which are connected, $R_{i\infty}$ and $R_{j\infty}$ are real parts of subsystem's impedances. In Eq. (6), infinite system impedances are used since dynamics of finite systems converge to semi-infinite/infinite systems dynamics at high frequencies. Moment and force impedance of an infinite plate can be expressed as:

$$Z^F = 8\rho h\kappa c'_L, \tag{7}$$

$$Z^M = \frac{16\rho h\kappa c'_L \big/ k_B^2}{1+1j(4/\pi)\ln(1/k_B r)}, \tag{8}$$

where k_B is bending wave number and r radius of excitation.

2.2. Power injection method (PIM)

2.2.1. Damping loss factor (DLF) of a single subsystem

DLF can be determined by different experimental methods such as decay rate and power injection. The power injection method is much more convenient if the damping of a subsystem is very sensitive to frequency. The DLF can be written for a single subsystem based on SEA power balance equation as [1]:

$$\eta = \frac{\left\langle \overline{P_{in}} \right\rangle}{\omega \left\langle \overline{E_{tot}} \right\rangle}. \tag{9}$$

Here, $E_{tot} = m\langle \overline{v}^2\rangle$ is total energy of vibrating subsystem ($E_{tot} \approx 2E_k$, E_k: kinetic energy), m is subsystem mass, $\langle \overline{v}^2\rangle$ is time and frequency-averaged mean square velocity of subsystem. The input power can be given as [1].

$$\left\langle \overline{P_{in}} \right\rangle = \langle \overline{Fv}\rangle = \left\langle F^2 \right\rangle \mathrm{Re}\left\{\left\langle \overline{Y_0} \right\rangle\right\} \tag{10}$$

where $\mathrm{Re}\{\langle \overline{Y_0}\rangle\}$ is the real part of time- and frequency-averaged excitation point mobility and F is the excitation force having constant spectral amplitude.

2.2.2. In situ coupling and damping loss lactors (CLF-DLF)

For *in situ* determination of loss factors, PIM is based on the determination of random power injected to subsystems and the measurement of total vibrational energies of those subsystems. In this technique, first, the power is injected to the first subsystem and then total energy of each subsystem is measured. Normalized form of the balance equation between S number of subsystems can be written in terms of total loss quantities (factors) $\tilde{\eta}_{ij}$:

$$\begin{bmatrix} \tilde{\eta}_{11} & \tilde{\eta}_{12} & \cdots & \tilde{\eta}_{1S} \\ \tilde{\eta}_{21} & \tilde{\eta}_{22} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\eta}_{S1} & \cdots & \cdots & \tilde{\eta}_{SS} \end{bmatrix} \begin{Bmatrix} E_{11}^n \\ E_{21}^n \\ \vdots \\ E_{S1}^n \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{Bmatrix} \tag{11}$$

where E_{ij}^n is the normalized vibrational energy, and is defined as