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Non-linear buckling analysis of FGM toroidal shell segments filled inside by an elastic medium under external pressure loads including temperature effects

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ABSTRACT

The analytical approach is presented to investigate non-linear buckling analysis and post-buckling behavior of FGM toroidal shell segments filled inside by an elastic medium under external pressure loads including temperature effects. The governing equations of non-linear buckling of FGM toroidal shell segments are derived based on the classical thin shell theory with the geometrical nonlinearity in von Karman–Donnell sense, Stein and McElman assumption and elastic medium modeled by Pasternak's two-parameter elastic foundation. The static critical buckling loads and the post-buckling pressure–deflection curves in two cases: movable and immovable boundary conditions including temperature effects are obtained by using the Galerkin's method. In the paper, the results are also compared with the solutions published in the literature for the specific cases. Effects of geometrical and material parameters, elastic foundation and temperature on the nonlinear buckling behavior of shells are shown in obtained results.

1. Introduction

The terminology of functionally graded materials (FGMs) was first discovered by Japanese scientists in the area of Sendai in 1984 [1,2]. Nowadays, in the world, the static and dynamic problems of FGM shell always attract the attention of many scientists. In the previous studies, D. Hui et al. [3,4] showed the delamination and energy absorbed of graphite/epoxy composites in fracture increase with impact energy. Furthermore, Sofiyev [5] investigated the buckling of composite orthotropic truncated conical shell under a combined axial compression and external pressure on elastic foundation using the superposition and Galerkin methods. In the other hand, mechanical and physical properties of FGMs are better than the traditional fiber-reinforced and laminated composite materials such as avoiding the stress concentration and delamination, oxidation resistance, high toughness, and heat-resistance. Therefore, FGMs are applied to heat-resistant, lightweight structures as aerospace industry, mechanical and medical industry. Due to improving requires for heat-resistant structures as well as the different medium, the studies about FGM structures including FGM plates and FGM shells have drawn much interest.

The post-buckling analysis of FGM cylindrical shells under axial compression and thermal loads using the element-free kp-Ritz method was presented by Liew [6]. The nonlinear bending and postbuckling of FGM cylindrical panels under combined loadings and resting on elastic foundations in thermal environment were analyzed by Shen and Wang [7]. Based on a higher order shear deformation theory and von Karman strain displacement relationships, the governing equations were solved by a singular perturbation technique along with a two-step perturbation approach. The influence of a Pasternak elastic foundation on the stability of FGM cylindrical shells under hydrostatic pressure based on the first order shear deformation theory considering the shear stresses was investigated by Najafov et al. [8]. Sofiyev and Kuruoglu [9] studied the nonlinear buckling of the truncated conical shell made of functionally graded materials surrounded by an elastic medium using the large deformation theory with von Karman–Donnell type of kinematic nonlinearity. By using the superposition and Galerkin methods, the nonlinear stability equations for the FGM truncated conical shell were solved. Hui-Shen Shen [10] investigated postbuckling analysis of axially-loaded

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functionally graded cylindrical shells in thermal environments using the classical shell theory with von Karman–Donnell-type of kinematic nonlinearity. The nonlinear torsional buckling and postbuckling of eccentrically stiffened FGM cylindrical shells in thermal environment based on the classical shell theory with von Karman geometrical nonlinearity and smeared stiffeners technique were investigated by Dung and Hoa [11]. Furthermore, the analytical approach to investigate the nonlinear axisymmetric response of functionally graded shallow spherical shells subjected to uniform external pressure incorporating the effects of temperature was given by Bich and Tung [12]. Shariyat [13] pointed out the dynamic buckling of imperfect FGM cylindrical shells with integrated surface-bonded sensor and actuator layers subjected to some complex combinations of thermo-electro-mechanical loads. The general form of Green's strain tensor in curvilinear coordinates and a high-order shell theory proposed earlier by Shariyat was used.

Huang and Han [14] investigated the nonlinear postbuckling behaviors of functionally graded cylindrical shells under uniform radial pressure using the nonlinear large deflection theory of cylindrical shells with the temperature-dependent material properties taken into account. With the same author [15], the nonlinear buckling and postbuckling of heat functionally graded cylindrical shells under combined axial compression and radial pressure were researched. In the analysis, the nonlinear strain–displacement relations of large deformation and the Ritz energy method were used while by taking the temperature-dependent material properties into account; various effects of external thermal environment were also investigated. Furthermore, Huang and Han [16] gave the study about nonlinear dynamic buckling of functionally graded cylindrical shells subjected to time-dependent axial load. The nonlinear dynamic equilibrium equation of the shell was achieved by applying an energy method and was then solved using the four-order Runge–Kutta method. The nonlinear buckling and postbuckling behavior of functionally graded stiffened thin circular cylindrical shells subjected to external pressure were studied by Dung and Hoa [17]. Fundamental relations, equilibrium equations are derived based on the smeared stiffeners technique and the classical shell theory with the geometrical nonlinearity in von Karman sense. Bich et al. [18] presented an analytical approach to investigate the linear buckling of truncated conical panels made of functionally graded materials and subjected to axial compression, external pressure and the combination of these loads. Moreover, Bich et al. [19] researched the nonlinear static and dynamic buckling of imperfect eccentrically stiffened functionally graded thin circular cylindrical shells subjected to axial compression based on the classical thin shell theory. The research on nonlinear torsional buckling and postbuckling of eccentrically stiffened functionally graded thin circular cylindrical shells was given by Dung and Hoa [20]. Sofiyev [21] discussed the nonlinear buckling behavior of truncated conical shells made of FGM using the large deformation theory with the von Karman–Donnell-type of kinematic non-linearity subjected to a uniform axial compressive load. Recently, Sofiyev [22] investigated the vibration and stability of FGM conical shells under a compressive axial load using the shear deformation theory based on Donnell shell theory and solved by Garlerkin method. The study achieved closed-form solutions for the dimensionless frequencies and critical axial loads for freely-supported FG truncated conical shells on the basic of the shear deformation theory.

The structures on the elastic medium have been developed for a long time by a lot of scientists. The simplest model for the elastic foundation is Winkler [23] like a series of separated springs without coupling effects between each other and then the model added a shear layer to one-parameter model is enhanced by Pasternak [24]. Sofiyev [25,26] studied the buckling of FGM shells on elastic foundation. Moreover, the postbuckling of FGM cylindrical shells surrounded by an elastic medium was presented by Shen [27,28]. Bagherizadeh et al. [29] investigated the mechanical buckling of functionally graded material cylindrical shells surrounded by Pasternak elastic foundation. Theoretical formulations were presented based on a higher-order shear deformation shell theory. The free vibration characteristics of FGM cylindrical shells partially resting on elastic foundation with an oblique edge by an analytical method based on the first order shear deformation theory were investigated by Kim [30]. Shen and Wang [31] studied the large amplitude vibration behavior of a shear deformable FGM cylindrical panel resting on elastic foundations in thermal environments with two kinds of micromechanics models: Voigt model and Mori–Tanaka model. The motion equations were based on a higher order shear deformation shell theory including shell panel–foundation interaction and solved by a two step perturbation technique to determine the nonlinear frequencies of the FGM cylindrical panel.

Toroidal shell segment has been used in such applications as satellite support structures, fusion reactor vessels, rocket fuel tanks, diver's oxygen tanks and underwater toroidal pressure hull. Some components of the above-mentioned structures may be made of FGM. The homogenous and isotropic toroidal shell segments were investigated by Stein and McElman [32] about the buckling problem. McElman [33] carried out the eccentrically stiffened shallow shells of double curvature with the static and dynamic behaviors. Furthermore, the initial post-buckling behavior of toroidal shell segments subject to several loading conditions based on the basic of Koiter's general theory was studied by Hutchinson [34]. Parnell [35] pointed out a simple technique for the analysis of shells of revolution applied to toroidal shell segments. Moreover, the influence of extra term on asymptotic analysis of imperfection sensitivity of toroidal shell segment with random imperfection was presented by Oyesanya [36]. Recently, there have been some new publications about toroidal shell segment structures. Bich et al. [37] studied the buckling of eccentrically stiffened functionally graded toroidal shell segment under axial compression, lateral pressure and hydrostatic pressure based on the classical thin shell theory, the smeared stiffeners technique and the adjacent equilibrium criterion. Furthermore, the nonlinear buckling and post-buckling problems of ES-FGM surrounded by an elastic medium under torsional load based on the analytical approach are investigated by Bich et al. [38].

To best of the authors' knowledge, there is no analytical approach on the nonlinear buckling of FGM toroidal shell segments subjected to external pressure in thermal environment.

In the present paper, the nonlinear buckling and postbuckling behaviors of FGM toroidal shell segments filled inside by an elastic medium under external pressure loads including temperature effects are considered. The governing equations are derived based on the classical shell theory with the nonlinear strain–displacement relation of large deflection. The three-term solution of deflection is used and by using Galerkin's method the closed-form expressions to determine critical buckling load and nonlinear post-buckling loads–deflection curves are obtained. The effects of buckling modes, volume fraction index, elastic foundation and thermal environment on the nonlinear buckling behavior of shells are meticulously considered.

2. Governing equations

2.1. Functionally graded material (FGM)

FGMs are microscopically inhomogeneous materials, in which material properties vary smoothly and continuously from one surface to the other structure surface. These materials are made