Composites Part B 87 (2016) 149–160

Contents lists available at ScienceDirect

Composites Part B

journal homepage: www.elsevier.com/locate/compositesb

A simple-input method to analyze thick composite tubes under pure bending moment reinforced by carbon nanotubes

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ARTICLE INFO

Article history:
Received 11 March 2015
Received in revised form
27 August 2015
Accepted 16 October 2015
Available online 2 November 2015

Keywords:
A. Glass fibres
B. Strength
C. Analytical modelling
C. Finite element analysis (FEA)
D. Mechanical testing

ABSTRACT

In the present work, thick laminated carbon nanotube-reinforced composite straight tubes subjected to pure bending moments are investigated using a new simple-input analytical method. The most general displacement field of elasticity for an arbitrary laminated orthotropic tube is employed to analytically determine stresses under pure bending moments based on a layer-wise method. The accuracy of the proposed method is subsequently verified by comparing the numerical results obtained using the proposed method and finite element method (FEM) with experimental data. The results show good agreement. Also, high efficiency in terms of computational time is achieved when the proposed method is used as compared with FEM. In addition, effects of using nanotubes in laminated composites on stress distributions of orthotropic straight tubes are investigated.

1. Introduction

Composite tube structures are frequently used in aerospace applications. Therefore, prediction of the state of stress in different layers of composite tubes is of theoretical interest and practical importance. In all applications, accurate design and inclusive analysis are important to ensure safety and proper performance of composite structures. It should be noted that stress analysis of cylindrical composite structures is often a complex task. A few reasons exist for such a complexity. First, the governing equations of composite tubes are complicated. Second, a major source of intricacy is the layerwise failure of composite materials. In fact, as soon as a layer fails, a delamination happens or a crack propagates inbetween the plies, material properties degrade and sometimes the governing equations could be different. Moreover, the tube geometry is a lot more complicated than flat geometry.

The researchers have performed a lot of investigations on composite tubes under different types of loading. Lekhnitskii [1] developed the solution for composite cylinder under bending load by using the system of partial differential equations. Kollár and Springer [2] performed a stress analysis on thin to thick-walled composite cylinders under hydrothermal and mechanical loads. Jolicoeur and Cardou [3] developed a general analytical solution in order to find the stresses and displacements fields of a composite cylinder subjected to bending, tensile and torsion loads. Cylinders made of functionally graded materials (FGM) under tension and bending were analysed [4]. An analysis on a cylindrically anisotropic elastic body was made when the body was subjected to extension, torsion, bending and thermo-mechanical [5]. Huang [6] developed the ultimate bending response of a solid composite cylinder reinforced with uniaxially continuous fibers. Fatigue behavior of unidirectional glass fiber reinforced polyester composites under in-phase combined torsion/bending loading was investigated [7]. A formulation of Generalized Beam Theory was derived to analyze the non-classical effects on the structural behavior of FRP composite circular hollow sections [8]. The effects of inner and outer reinforcements on the bending behavior of a thin walled tube were studied [9]. The stress analysis of hollow composite cylindrical structures subjected to different loads was performed [10]. Their method was efficient for thin-walled hollow composite tubes. Based on the nonlinear ring theory, mechanical behavior of thermoplastic tube under combined bending and tension was investigated [11]. They verified formulations with FEM results obtained using ABAQUS. Menshykova and Guz [12] performed a stress analysis on laminated thick composite pipes subjected to bending loading. They found stresses as a function of the material properties, thickness, lay-up and the magnitude of bending load. A model were developed to study the failure of 0° laminated composite tubes under static and fatigue loadings [13]. Wang et al. [14] analyzed the behavior of luffa-filled tubes

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http://dx.doi.org/10.1016/j.compositesb.2015.10.025
1359-8368/

under uniaxial compression numerically using and analytically using finite element analysis and theoretical models, respectively. In addition, they validated FEA models against experimental data. Gohari et al. [15] used first-ply failure to study a laminated glass fiber reinforced polymer composite shell subjected to internal pressure. Capela et al. [16] investigated the fatigue behaviour of composite tubes under bending/torsion dynamic loadings. Recently, static analysis of carbon nanotube-reinforced composite cylinder under thermo-mechanical was studied using Mori–Tanaka theory [17]. Recently, a method to analyze stress distributions of the composite cantilever straight tube was developed [18].

Although finite element and other methods reviewed above can be used for analysing tube structures, it is necessary to do some actions for each structure every time some dimensions are changed. Therefore, it is desired to have a method where the input for the solution is simple; i.e. one only needs to enter the actual dimensions without meshing work. The present work is devoted to develop an analytical method that can provide stresses, strains and deformations for a thick composite tube subjected to pure bending moment with simple inputs. The layer-wise method, which includes the full three-dimensional constitutive relations, is employed to calculate the three-dimensional stress distributions within the tube. Then, the comparison is made between results obtained using the proposed analytical method, experimental data and FEM (ANSYS). Finally, effects of reinforcing composite tubes with multi-walled carbon nanotubes (MWCNTs) on stress distributions are studied.

2. Theoretical Formulation

2.1. Displacement Field

A thick laminated orthotropic straight tube with mean radius R and thickness h is subjected to bending moment M_0 as shown in Figure 1. The cylindrical coordinates (x, θ, r) are placed at the middle of the composite tube where x and r are the axial and radial coordinate, respectively. The integration of the appropriate linear strain-displacement relations of elasticity, within cylindrical coordinate system will yield the following displacement field for the kth layer:

$$u_1^{(k)}(x,\theta,r) = xr\left(C_5^{(k)}\cos\theta + C_4^{(k)}\sin\theta\right) + C_6^{(k)}x + u^{(k)}(\theta,r) \tag{1a}$$

$$u_2^{(k)}(x,\theta,r) = x\left(C_1^{(k)}\cos\theta - C_2^{(k)}\sin\theta - C_3^{(k)}r\right) - \frac{1}{2}x^2\left(C_4^{(k)}\cos\theta - C_5^{(k)}\sin\theta\right) + v^{(k)}(\theta,r) \tag{1b}$$

$$u_3^{(k)}(x,\theta,r) = x\left(C_1^{(k)}\sin\theta + C_2^{(k)}\cos\theta\right) - \frac{1}{2}x^2\left(C_5^{(k)}\cos\theta + C_4^{(k)}\sin\theta\right) + w^{(k)}(\theta,r) \tag{1c}$$

where $u_1^{(k)}(x,\theta,r)$, $u_2^{(k)}(x,\theta,r)$ and $u_3^{(k)}(x,\theta,r)$ represent the displacement components in the x, θ and r directions, respectively, of a material point located at (x, θ, r) in the kth ply of the laminated composite tube in Figure 1. Also, $u^{(k)}(\theta,r)$, $v^{(k)}(\theta,r)$ and $w^{(k)}(\theta,r)$ (k=1, 2, …, N+1) represent the displacement components of all points located on the kth layer in the undeformed laminated tube. In order to satisfy the interfacial continuities of the displacement components, it is necessary that the integration constants appearing in Eqs. (1) to be the same for all layers. Thus, Eqs. (1) are represented as:

$$u_1^{(k)}(x,\theta,r) = xr(C_5\cos\theta + C_4\sin\theta) + C_6x + u^{(k)}(\theta,r) \tag{2a}$$

$$u_2^{(k)}(x,\theta,r) = x(C_1\cos\theta - C_2\sin\theta - C_3r) - \frac{1}{2}x^2(C_4\cos\theta - C_5\sin\theta) + v^{(k)}(\theta,r) \tag{2b}$$

$$u_3^{(k)}(x,\theta,r) = x(C_1 in\theta + C_2\cos\theta) - \frac{1}{2}x^2(C_5\cos\theta + C_4\sin\theta) + w^{(k)}(\theta,r) \tag{2c}$$

Moreover, in Eq. (2a), $u^{(k)}(\theta,r)$ can be replaced by $-C_1 r\sin\theta + u^{(k)}(\theta,r)$. It can be shown that the terms involving C_1 in Eq. (2) correspond to an infinitesimal rigid-body rotation. These terms will, therefore, be ignored in the following developments since they will not generate any strain. Similarly, it can be readily shown that the terms involving C_2 must also be eliminated since they represent another rigid-body rotation of the tube. Furthermore, as long as the loading conditions at the two ends of the tube are identical, the constant C_4 must vanish in order to satisfy the symmetry condition in deformation $u_3^{(k)}(x,\theta,r) = u_3^{(k)}(-x,-\theta,r)$. It is thus concluded that the most general form of the displacement field for the kth layer of a composite tube is given as:

$$u_1^{(k)}(x,\theta,r) = C_5xr\cos\theta + C_6x + u^{(k)}(\theta,r) \tag{3a}$$

$$u_2^{(k)}(x,\theta,r) = -C_3rx + \frac{1}{2}C_5x^2\sin\theta + v^{(k)}(\theta,r) \tag{3b}$$

$$u_3^{(k)}(x,\theta,r) = -\frac{1}{2}C_5x^2\cos\theta + w^{(k)}(\theta,r) \tag{3c}$$

2.2. Layerwise Theory (LWT)

The equivalent single-layer theories are unable to precisely represent the local phenomena in laminated composites, like stress and strain distributions. But then, the LWTs, which consider real 3-D behavior of each layer, are able to present accurate results considering the localized phenomena. Many investigations on the use of LWT for analyzing composite structures were performed [19, 20]. In LWT, the displacement components of a generic point in the laminate are conveniently given as:

$$u_1(x,\theta,z) = u_i(x,\theta)\Phi_i(z) \tag{4a}$$

$$u_2(x,\theta,z) = v_i(x,\theta)\Phi_i(z) \tag{4b}$$

$$u_3(x,\theta,z) = w_i(x,\theta)\Phi_i(z) \quad (i = 1,2,\ldots,N+1) \tag{4c}$$

where k, here and in what follows, being a dummy index implying summation of terms from i=1 to i=N+1. In Eqs. (4), u_1, u_2 and u_3 denote the displacement components in the x, θ and r directions, respectively. Also, $u_i(x,\theta)$, $v_i(x,\theta)$ and $w_i(x,\theta)$ represent the displacements of the points initially located on the ith surface of the laminated tube in the x, θ and r directions, respectively.

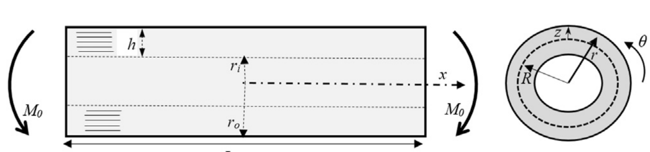

Figure 1. The geometry of a composite straight tube and the coordinate system.