Composites Part B 87 (2016) 161–175

Contents lists available at ScienceDirect

Composites Part B

journal homepage: www.elsevier.com/locate/compositesb

Nonlinear bending analysis of bilayer orthotropic graphene sheets resting on Winkler–Pasternak elastic foundation based on non-local continuum mechanics

Shahriar Dastjerdi*, Mehrdad Jabbarzadeh

Department of Mechanical Engineering, Mashhad Branch, Islamic Azad University, Mashhad, Iran

ARTICLE INFO

Article history:
Received 16 March 2015
Received in revised form
20 September 2015
Accepted 6 October 2015
Available online 31 October 2015

Keywords:
A. Nano-structures
A. Plates
B. Elasticity
C. Numerical analysis

ABSTRACT

In this paper, the nonlinear bending behavior of bilayer orthotropic rectangular graphene sheets resting on a two parameter elastic foundation is studied subjected to uniform transverse loads using the non-local elasticity theory. The non-local theory consider the small scale effects. Considering the non-local differential constitutive relations of Eringen theory based on first order shear deformation theory (FSDT) and using the von-Karman strain field, the nonlinear formulations are derived. Equilibrium partial differential equations are expressed in terms of generalized displacements and rotations. Because of nonlinear partial differential equations, if it is not impossible but it is too complicated to find an analytical solution, so, the differential quadrature method (DQM) that is a high accurate numerical method is investigated to solve the governing equations. The Newton–Raphson iterative scheme is applied to solve the obtained nonlinear algebraic equations system. Different boundary conditions including clamped, simply supports and free edges are considered. Since there is not any researches available for nonlinear bending of bilayer rectangular graphene sheets with FSD theory, so considering the monolayer, the results are compared with available papers. Finally, the small scale effect parameter due to various types of conditions such as thickness ratio, boundary conditions, stiffness of elastic foundation, the van der Waals interactions between the layers, nonlinear to linear FSDT analysis and the differences between non-local and local elasticity theories are investigated.

1. Introduction

Nowadays, nano structures are used widely such as nanotubes, nano beams and nano plates. The graphene sheets are kind of nano materials which are formed in hexagonal shape by covalent bonds between carbon atoms. Especial properties of graphene sheets such as high strength, the low ratio of weight to area unit and extraordinary electrical properties, attracted many researchers to consider this topic as their major activities [1–3]. It is studied the possibility of using graphene in cellular photographing [4], the mass sensors and monitoring of atomic dusts [5], composite materials [6,7], the gas detectors [8] and in microelectronic and biomechanics sets [9]. The bending strength of graphene sheet is low, so using multilayers of graphene sheets causes to improve this weakness. In order to make multilayers of graphene sheet, several single layers of graphene are set on each other by weak van der Waals bond between the surface atoms [10].

There are different methods to analyze the nano structures [11]. Except experimental methods, it can be mentioned atomic modeling [12], combination of atomic modeling and continuum mechanics [13] and continuum mechanics [14]. Inasmuch as the control of experimental and atomic modeling is difficult and expensive in computations; consequently, the continuum mechanics method is attended by many researchers, because of convenience in formulations and acceptable results in comparison with two other methods [15]. The continuum mechanics method is categorized to three different methods: 1- couple stress theory [16], 2- modified strain gradient theory [17], and 3-The Eringen non-local elasticity theory [18]. The Eringen non-local elasticity theory is widely used to analyze the mechanical behavior of nano-structures. Eringen theory by considering the small scale effects, explains that the stress in a reference point is affected by the strains in whole body domain or the interactive bonds between the carbon atoms are not neglected and have significant effects on mechanical

* Corresponding author. P.O.B. 9185734943 Mashhad, Iran.
E-mail address: dastjerdi_shahriar@yahoo.com (S. Dastjerdi).

http://dx.doi.org/10.1016/j.compositesb.2015.10.018
1359-8368/

behavior in nano scales. So, using of classical elasticity theory generates unacceptable results [19].

Reddy et al. [20], reformulated the different types of beam theories such as Euler–Bernoulli, Timoshenko, Levinson and Reddy based on Eringen non-local elasticity theory and studied the bending, vibrations and buckling of the nano beams. Pradhan [21] investigated the buckling of rectangular graphene sheets, considering the isotropic material properties and the third order shear deformation theory (TSDT). They showed that using of TSDT plate theory for moderately thick plates causes the more accurate results. Shen Shen [22] studied bending, vibrations and post buckling of rectangular graphene plates resting on elastic foundation, using classical plate theory by considering the nonlinear strains field in thermal environment. Ansari et al. [23] proposed an analytical solution to calculate the critical buckling load for a mono layered graphene sheet under uniform loading by use of Galerkin method. Hosseini-hashemi et al. [24] investigated the buckling of rectangular graphene plates using the Mindlin and Eringen non-local elasticity theories. The obtained results are compared with Euler–Bernoulli, CLPT and higher order shear deformation theories. Pouresmaeeli et al. [25] obtained the exact solution for non-local vibration of double-orthotropic nanoplates embedded in elastic medium. Jomehzadeh and Saidi [26] studied the large amplitude vibrations of bilayer graphene sheets embedded in a nonlinear polymer matrix. Mohammadi et al. [27] studied free transverse vibration analysis of circular and annular graphene sheets using non-local continuum mechanics for various types of boundary conditions. Zhou et al. [28] studied the bending of bilayer graphene sheets under transverse loading in thermal environment. The nonlinear strains field and CLP theory are used in that literature and they showed that the small scale effects have an important role in nonlinear bending analysis of graphene sheets. Zenkour et al. [29] investigated the thermal buckling of nano plates embedded in an elastic Winkler–Pasternak matrix, using the sinusoidal shear deformation plate theory and compared the results with CLPT and FSDT theories. Radebe et al. [30] studied the buckling of rectangular nano-plates with uncertain orthotropic material properties using non-local theory. They considered the nano-plate as a non-local plate to take the small-size effects into account with the small-scale parameter also taken to be uncertain. They studied the effect of small scale on natural frequencies. Shear buckling of orthotropic rectangular graphene plates in thermal environment is investigated by Mohammadi et al. [31]. They showed that the critical shear buckling load of single layers graphene sheets is strongly dependent on the small scale parameter. The Exact closed-form solution for non-local vibration and biaxial buckling of bonded multi-nanoplate system is presented by Karličić et al. [32]. The nonlinear bending analysis of mono layered rectangular graphene sheets in an elastic matrix is investigated by Golmakani and Rezatalab [33]. They proved that the maximum deflection decreases along the increase of non-local parameter. Also in that paper, the obtained results for local and non-local elasticity theories are compared with each other.

In present study, the nonlinear bending of a bilayer orthotropic graphene sheet is investigated based on the first-order shear deformation theory considering Von Karman strain field. In order to study the small scale effects on deflection, the Eringen's non-local theory is applied. The DQM which is a numerical method is applied to solve the partial differential governing equations. Because, there are not any literatures available on bending of bilayer graphene sheets, in order to demonstrate the accuracy of obtained results, the results are validated with available papers in bending of mono layer graphene sheets. The effects of different conditions such as variation of van der Waals interaction bonds between the layers, thickness of the graphene nanoplate, the length to width ratio, non-local to local deflection ratio, linear to nonlinear analysis, different types of boundary conditions, and the effects of non-local parameter and the value of Winkler–Pasternak matrix on the results are investigated.

2. Governing equations

A bilayer rectangular graphene sheet is considered with thickness h, the length L_x, the width L_y, under uniform transverse loading q resting on an elastic Winkler–Pasternak matrix. The geometry of the plate is shown in Fig. 1. Most of the researches are based on the classical plate theory (CLPT). This theory is only acceptable for the plates that the ratio of thickness to length is small and includes some assumptions; for example, neglecting the effects of transverse shear deformations. When the thickness to length ratio is considerable, the effects of transverse shear deformations are significant and must be considered. Assuming that the ratio of thicknesses to length in graphene sheets is significant, so in this paper, all the governing equations are derived based on the first-order shear deformation theory (FSDT) that considers the neglected assumptions in classical plate theory. According to the first-order shear deformation theory, the displacement field can be expressed as:

$$Ui(x,y,z) = ui(x,y) + z\psi i_1(x,y) \quad (i = 1,2) \tag{1}$$

$$Vi(x,y,z) = vi(x,y) + z\psi i_2(x,y) \quad (i = 1,2) \tag{2}$$

$$Wi(x,y,z) = wi(x,y) \quad (i = 1,2) \tag{3}$$

In Equations (1)–(3), ui, vi and wi are the displacement components of the mid-plane along the x, y and z directions, respectively. ψi_1 and ψi_2 explain the rotation functions of the transverse normal about y and x directions. The index i = 1, 2 refers to upper and bottom layers respectively. The graphene sheet is assumed to have large deformation, so considering von-Karman assumptions, the strain field are expressed as follows:

$$\varepsilon i_x = \frac{\partial ui}{\partial x} + z\frac{\partial \psi i_1}{\partial x} + \frac{1}{2}\left(\frac{\partial wi}{\partial x}\right)^2 \quad (i = 1,2) \tag{4}$$

$$\varepsilon i_y = \frac{\partial vi}{\partial y} + z\frac{\partial \psi i_2}{\partial y} + \frac{1}{2}\left(\frac{\partial wi}{\partial y}\right)^2 \quad (i = 1,2) \tag{5}$$

$$\gamma i_{xz} = \frac{\partial wi}{\partial x} + \psi i_1 \quad (i = 1,2) \tag{6}$$

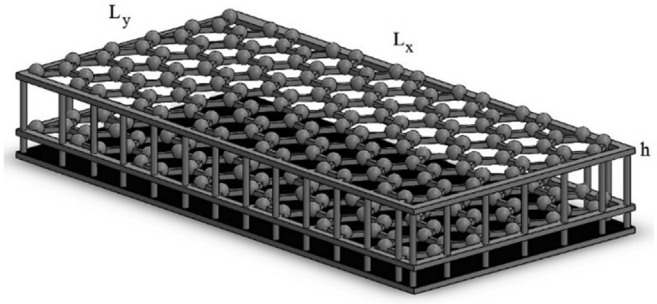

Fig. 1. Geometry of bilayer rectangular graphene sheet.