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# Elasticity solution of functionally graded carbon nanotube-reinforced composite cylindrical panel subjected to thermo mechanical load

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ABSTRACT

Thermoelastic analysis of composite cylindrical panel reinforced by single walled carbon nanotube (SWCNT) with simply supported edges by using three-dimensional theory of elasticity. Thermoelastic constant of carbon nanotube (CNT) as well as polymer matrix are assumed to be temperature independent. The volume fractions of oriented, straight SWCNTs are assumed to be uniformly distributed (UD) and or graded in the thickness direction according to four kinds of CNT distributions. The effective material properties of the nanocomposite cylindrical panel are based on rule of mixture. At first temperature distribution in three dimensions is obtained by solving heat conduction differential equation with variable coefficient. By applying Fourier series expansion to the stress and displacement fields along the axial and circumferential direction and state space technique along the radial direction thermoelastic analysis is carried out. Moreover, effects of volume fraction of carbon nanotube, uniform distribution and functionally graded distribution of CNT, mid radius to thickness ratio, length to mid radius ratio, thermal and mechanical surface boundary conditions on bending behavior of FG-CNTRC cylindrical panel are also examined.



## 1. Introduction

Recently interesting has been growing continuously towards CNT due to its exceptional mechanical, thermal and electrical properties. These properties causes the CNT to be considered as significant reinforcement materials and to become the building blocks of a new generation of composite materials for high performance composite structures with much application potential [1,2]. Due to these properties of CNT, the study on the mechanical and thermal properties of CNTRC beam, plate and shell structures has been carried out by many researchers in recent years. By using the molecular dynamics (MD) simulations and experimental method, Gou et al. [3] investigated the interfacial bonding of SWCNT reinforced epoxy composites. Wuite and Adali [4] carried out a multi-scale analysis of the deflection and stress behavior of CNT reinforced polymer composite beams. Vodenitcharova and Zhang [5] investigated pure bending and bending-induced local buckling of CNTRC beams reinforced by single walled CNT. Shen [6] discussed nonlinear bending behavior of simply supported, functionally graded composite plates reinforced by SWCNTs under transverse uniform or sinusoidal load in thermal environments. By using a multi-scale approach, Shen and Zhang [7] discussed thermal buckling and post-buckling behaviors of functionally graded nanocomposite plates reinforced by SWCNTs subjected to in-plane temperature variation. Shen [8] investigated post-buckling behavior of nanocomposite cylindrical shells reinforced by SWCNTs subjected to axial environments by using the higher order shear deformation theory as well as a von Kármán-type of kinematic nonlinearity. Based on a micromechanical model and multi-scale approach, Shen [9] discussed post-buckling behavior of FG-CNTRC cylindrical shells subjected to mechanical load in thermal environments. Based on a higher order shear deformation plate theory, nonlinear vibration of FG-SWCNT plates rested on elastic foundation in thermal environments was investigated by using an improved perturbation technique [10]. Based on the first-order shear deformation (FSDT) and using Mindlin plate theory, mechanical buckling behavior of FG nanocomposite plate reinforced by SWCNTs was discussed by Mehrabadi et al. [11]. Zhu et al. [12] carried out bending and free vibration analyses of composite plates reinforced by SWCNTs by using the finite element method based on the first order shear deformation plate theory. Wang and Shen [13] investigated nonlinear bending and vibration behavior of

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**Nomenclature**

$E_{11}^{CNT}$, $E_{22}^{CNT}$, $G_{12}^{CNT}$, $E_m$, $G_m$ Young's modulus, shear modulus of carbon nanotube and matrix, respectively
$\alpha_{ij}^{CNT}$, $k_{CNT}$, $\alpha_m$, $k_m$ thermal expansion coefficient and thermal conductivity coefficient of carbon nanotube and matrix respectively
$k$ thermal conductivity coefficient of carbon nanotube reinforced composite
$L$, $h$ length and thickness of panel
$n$, $m$ number of half waves in $\theta$- and $z$ -direction
$P$ aspect ratio of CNTs, $L/d$ (length to diameter ratio of CNTs)
$R_k$ interface thermal resistance between CNT and matrix
$r_i$, $r_o$ inner and outer radius of cylindrical panel
$V_{CNT}$, $V_m$ carbon nanotube and matrix volume fractions, respectively
$W_{CN}$, $\rho_{CNT}$ mass fraction and density fraction of CNT, respectively
$\beta_i$ $(i = r,\theta,z)$ thermoelastic constants in $r$-, $\theta$-, $z$-direction, respectively
$\alpha_i$ $(i = r,\theta,z)$ thermal expansion coefficient in $r$-, $\theta$-, $z$-direction, respectively
$\rho_m$ density fraction of matrix
$\theta_m$ span angle of cylindrical panel
$\eta_i$ $(i = 1,2,3)$ CNT efficiency parameters accounting for the scale-dependent material properties
$u_r$, $u_\theta$, $u_z$ displacement components in $r$-, $\theta$-, $z$-directions, respectively
$\sigma_i$ $(i = r,\theta,z)$ normal stresses
$\tau_{r\theta}$, $\tau_{\theta z}$ $\tau_{rz}$ shear stresses
$\gamma_{z\theta}$, $\gamma_{rz}$, $\gamma_{r\theta}$ shear strains
$\varepsilon_i$ $(i = r,\theta,z)$ normal strains
$\delta$ state variables

sandwich plate with CNTRC face sheets by using the multi-scale approach and two-step perturbation technique. By using higher order shear deformation theory and Von Karman type of kinematic nonlinearity, Shen [14] discussed postbucking of FG-CNTRC cylindrical panel in thermal environment. Nonlinear vibration of FG-CNTRC cylindrical shell was investigated by Shen and Xiang [15] using higher-order shear deformation theory with a Von Karman-type of kinematic nonlinearity. Alizada et al. [16] investigated bending behavior of a substrate coated by nanomaterials with vacancies subjected to uniform extension load. Joshi et al. [17] used 3-D hexagonal representative volume element (RVE) with short and straight CNTs to investigated effects of inclined carbon nanotubes on mechanical properties for nano-composites. By using Eshelby—Mori—Tanaka approach and two-dimensional differential quadrature method, free vibration analysis of CNTRC cylindrical panel was presented by Sobhani Aragh [18]. Moradi-Dastjerdi et al. [19] analyzed dynamic behavior of FG-CNTRC cylindrical shell subjected to impact load by making the use of mesh free method. Based on higher order shear deformation shell theory, Shen and Xiang [20] carried out post-buckling analysis of FG-CNTR cylindrical shell subjected to combined axial and radial loads in thermal environment with a von Kármán-type of kinematic nonlinearity. Based on Donnell's theory, Mosallaie Barzokie et al. [21] investigated nonlinear buckling behavior of composite cylindrical shell made on polyvinylidene using Hamilton's principle as well as harmonic differential quadrature method (HDQM). Bhardwaj et al. [22] investigated the nonlinear static and dynamic behavior of cross-ply CNTRC laminated plate by using the double Chebyshev series. Based on three-dimensional theory of elasticity, Alibeigloo [23,24] investigated bending behavior of functionally graded carbon nanotube-reinforced composite plate and cylindrical panel embedded in piezoelectric layers. Recently author [25] used three dimensional theory of elasticity and state space method to analysis of functionally graded carbon nanotube-reinforced composite (FG-CNTRC) rectangular plate with simply supported edges under thermo-mechanical loads. Based on theory of elasticity, author [26] used presented thermoelastic behavior of CNTRC simply supported rectangular plate embedded in piezoelectric layers.

Surveying the open literature shows that Thermo-elastic behavior of FG-CNTRC cylindrical panel has not been yet considered. In this work three dimensional analytical solution for CNTRC cylindrical panel with simply supported boundary conditions and subjected to thermo mechanical load is carried out by using Fourier series solution for the stresses and displacement components along the axial and circumferential direction and state space technique along the radial direction.

## 2. Basic equations

A CNT reinforced composite cylindrical panel with axial length, L span angle, $\theta_m$ inner and outer radius, $r_i$ and $r_o$, respectively is considered (Fig. 1). The SWCNT reinforcement is either uniformly distributed (UD) or functionally graded (FG) in the thickness direction with various cases of distribution. According to the rule of mixture and considering the CNT efficiency parameters, the effective mechanical properties, Poisson's ratio, $\nu_{12}$ and thermal expansion coefficient, $\alpha$ are assumed as [6].

$$E_{11} = \eta_1 V_{CNT} E_{11}^{CNT} + V_m E^m \tag{1a}$$

$$\frac{\eta_2}{E_{22}} = \frac{V_{CNT}}{E_{22}^{CNT}} + \frac{V_m}{E^m} \tag{1b}$$

$$\frac{\eta_3}{G_{12}} = \frac{V_{CNT}}{G_{12}^{CNT}} + \frac{V_m}{G^m} \tag{1c}$$

$$\nu_{12} = V_{CNT}\nu_{12}^{CNT} + V_m \nu^m \tag{1d}$$

$$\alpha_{11} = V_{CNT}\alpha_{11}^{CNT} + V_m \alpha^m \tag{1e}$$

$$\alpha_{22} = \left(1 + \nu_{12}^{CNT}\right) V_{CNT}\alpha_{22}^{CNT} + (1 + \nu_m) V_m \alpha^m - \nu_{12}\alpha_{11} \tag{1f}$$

The CNT and matrix volume fractions has following relation

$$V_{CNT} + V_m = 1 \tag{2}$$

According to Fig. 1, CNT volume fraction distribution across the thickness has the cases of $UD$, $FG - V$, $FG - \Lambda$, $FG - X$ and $FG - \diamond$, respectively as follow

$$V_{CNT} = V_{CNT}^* \tag{3a}$$