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Thermal buckling of annular microstructure-dependent functionally graded material plates resting on an elastic medium

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ABSTRACT

First, modified couple stress theory is extended in the presence of thermo-mechanical loading. To this end, the generalized form of Hamilton's principle as well as constitutive relations are derived in general curvilinear coordinates. Then using the developed formalism, the bifurcation-type buckling of heated annular plates composed of functionally graded materials (FGMs) and resting on an elastic foundation is analytically studied. The non-classical FGM plate model contains a material length scale parameter and can interpret size effect. The adjacent equilibrium criterion is employed to derive stability equations. Thermo-mechanical properties of FGM plates are assumed to be graded across the thickness direction according to a power law form. Various types of thermal loading including uniform temperature rise, linear temperature distribution and heat conduction across the thickness are considered. A parametric study is conducted to investigate the influences of the material length scale parameter, power law index, inner and outer radii and also elastic foundation coefficients on thermal stability characteristics of FGM plates. The results reveal the existence of bifurcation-type buckling for a certain type of boundary conditions in which case the buckling patterns are asymmetric. Furthermore, the material length scale parameter and the geometry of annular FGM plates are shown to be more influential.

1. Introduction

Having found a wide range of applications, functionally graded materials (FGMs) are inhomogeneous composites in which both compositional profile and properties vary smoothly in one or more preferred direction(s). The advantages of structures made of FGMs have necessitated more investigations on their behavior. Many researchers have studied the bending [1], linear and nonlinear vibration [2–6], dynamic stability [7,8], buckling [9,10] and postbuckling [11,12] of FGM structures on the basis of the conventional continuum theory in the past two decades. In recent years, FGMs have been concerned for their application in micro-electro-mechanical systems [13–16], where microscale structures play a prominent role. In such scales, however, the behavior of materials is experimentally observed to be closely associated to their microstructures, often called size effect, and a classical treatment is no longer adequate. The motivation of this work is then to incorporate size effect into FGMs under thermo-mechanical loading for their potential applications.

The inadequacy of classical continuum theory has been experimentally demonstrated such that in the order of microns and submicrons, microstructure-dependent size effect is inevitable [17–22]. This means that conventional continuum theory is not capable of explaining the behavior of materials in such scales. Although size dependency has been observed in the past 20 years, the theoretical discussion of nonclassical continuum theories may be traced back to early 1960s when the so called couple stress theory (CST) was proposed [23–25]. Few works are reported in literature to present the static and dynamic analysis of various structures using CST [26,27]. Modified couple stress theory (MCST) was then proposed by Yang et al. [28]. For an isotropic elastic material, MCST introduces a new constant, called material length scale parameter, in constitutive relations in addition to two classical constants. A review of literature implies that MCST have enjoyed great success thus far. Very recently, Ashoori and Mahmoodi [29] presented a detailed derivation of MCST in general curvilinear coordinates. Their results, including deformation measures, governing equations, boundary conditions and constitutive relations, are then simplified for two practical orthogonal curvilinear coordinates, i.e. cylindrical and spherical coordinates, in terms of physical components which are more convenient. It is worth

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mentioning that such formulation may be appropriately applied to a wide range of problems in which the use of curvilinear coordinates is unavoidable, such as problems of cylindrical and spherical cavity expansion, the analysis of asymptotic crack tip field, the interpretation of micro/nano indentation tests and also bending or twisting tests on small scales.

Reported works on static and dynamic responses of structures based on MCST are numerous. Among the primary works on this subject, Ma et al. [30–32], Tsiatas [33], Asghari et al. [34] and Akgoz and Civalek [35] developed beam and plate models for bending, vibration and buckling problems. Ashoori and Mahmoodi [36] employed extended Kantorovich method, which is an accurate approximate closed-form solution, to investigate static bending of microstructure-dependent plates. A study of MCST-based Mindlin plates is presented by Roque [37] in which a meshless method is employed to solve governing equations numerically. Nonlinear MCST-based third-order FGM plates are formulated by Reddy and Kim [38] and solved by Kim and Reddy [39] and Gao et al. [40]. Reddy and Berry [41] provided nonlinear model for axisymmetric bending of circular FGM plates based on MCST. Moreover, bending and free vibration of FGM plates and buckling of sigmoid FGM plates embedded in Pasternak elastic medium are studied by Thai and Kim [42] and Jung et al. [43], respectively. Employing Galerkin method in conjunction to pseudo-arc length continuation technique, Ghayesh et al. [44] presented nonlinear dynamics of microscale beams. Bending, buckling and free vibration of annular Mindlin FGM plates based on MCST are investigated by Ke et al. [45] and Ansari et al. [46].

The literature search indicates that the present work is the first attempt to provide a detailed derivation of MCST so as to interpret size dependency in FGMs under thermo-mechanical loading. To this end, the generalized form of Hamilton's principle and also constitutive relations are derived in general curvilinear coordinates. Employing the developed formalism, governing equations and boundary conditions of heated annular size-dependent FGM plates resting on an elastic foundation are extracted. The adjacent equilibrium criterion is then utilized to obtain stability equations. Each thermo-mechanical property of FGM plates is assumed to vary in the thickness direction based on a power law form. Three types of thermal loading including uniform temperature rise, linear temperature distribution and heat conduction through the thickness are considered. The influences of the material length scale parameter, power law index, inner and outer radii and also elastic foundation coefficients on the thermal stability characteristics of FGM plates are discussed in detail.

2. Preliminaries

Let $\mathbf{E}^3$ denote a three-dimensional Euclidean space, $\widehat{\boldsymbol{e}}^i = \widehat{\boldsymbol{e}}_i$ be an orthonormal basis and $\widehat{x}_i$ be the Cartesian coordinates of a point $\widehat{x} \in \mathbf{E}^3$. In addition, a three-dimensional vector space $\mathbb{R}^3$ defined over the real field is considered. The coordinates of a point $x \in \mathbb{R}^3$ are identified with x_i and $\partial_i := \partial/\partial x_i$. Let there be given two open subsets $\widehat{\Omega} \subset \mathbf{E}^3$, $\Omega \subset \mathbb{R}^3$ and an immersion $\boldsymbol{\Xi} = \Xi_i \widehat{\boldsymbol{e}}^i : \Omega \to \mathbf{E}^3$ such that $\boldsymbol{\Xi}(\Omega) = \widehat{\Omega}$. Then, the three coordinates x_i represent the curvilinear coordinates of $\widehat{x}$. Since $\boldsymbol{\Xi}$ is an immersion, the matrix

$$\nabla\boldsymbol{\Xi}(x) := \begin{pmatrix} \partial_1\Xi_1 & \partial_2\Xi_1 & \partial_3\Xi_1 \\ \partial_1\Xi_2 & \partial_2\Xi_2 & \partial_3\Xi_2 \\ \partial_1\Xi_3 & \partial_2\Xi_3 & \partial_3\Xi_3 \end{pmatrix}(x)$$

is defined over Ω and is invertible. Therefore, three linearly independent vectors $\boldsymbol{g}_i(x) \in \mathbb{R}^3$ defined as

$$\boldsymbol{g}_i(x) := \partial_i \boldsymbol{\Xi}(x) = \begin{pmatrix} \partial_i\Xi_1 \\ \partial_i\Xi_2 \\ \partial_i\Xi_3 \end{pmatrix}(x)$$

form a basis. Next, the covariant components of the metric tensor are introduced

$$g_{ij}(x) := \boldsymbol{g}_i(x)\cdot\boldsymbol{g}_j(x) = \left(\nabla\boldsymbol{\Xi}(x)^T\nabla\boldsymbol{\Xi}(x)\right)_{ij}$$

Since the vectors $\boldsymbol{g}_i(x)$ are linearly independent, the nine relations

$$\boldsymbol{g}^i(x)\cdot\boldsymbol{g}_j(x) = \delta^i_j$$

unambiguously define three linearly independent vectors $\boldsymbol{g}^i(x)$ which in turn define the contravariant components of the metric tensor

$$g^{ij}(x) := \boldsymbol{g}^i(x)\cdot\boldsymbol{g}^j(x)$$

Let $\mathfrak{B}$ be a body occupying $\widehat{\Omega}^{\mathfrak{B}} \subset \widehat{\Omega}$ of volume $\mathscr{V}$ bounded by the closed surface $\partial\mathscr{V}$, subjected to surface traction $\boldsymbol{t}^{[\boldsymbol{n}]}$ and surface couple $\boldsymbol{\mu}^{[\boldsymbol{n}]}$ vectors, both per unit area transmitted through the surface $\partial\mathscr{V}$ and also body force $\boldsymbol{F}$ and body couple $\boldsymbol{C}$ vectors, each per unit volume. According to MCST proposed by Yang et al. [28], governing equations are given by

$$\begin{aligned} & t^{ji}\,\|_j + F^i = \rho\ddot{u}^i \\ & \Im_{ijk}t^{jk} + \mu^j_{i\|j} + C_i = 0 \end{aligned} \tag{1}$$

with ρ being density, u^i the components of displacement vector, t^{ij} and μ^{ij} the components of asymmetric stress and symmetric couple stress tensors, respectively, $\Im_{ijk} = \sqrt{g}e_{ijk}$ the components of Levi-Civita tensor in which $g = \det g_{ij}$ and e_{ijk} are the components of Levi-Civita pseudo-tensor-relative tensor with the weight of -1, and the parallel lines show covariant derivative. Moreover, the variation of strain energy density is written as

$$\delta\mathscr{U} = \sigma^{ij}\delta\varepsilon_{ij} + \mu^{ij}\delta\chi_{ij} \tag{2}$$

where χ_{ij} are the components of a new strain measure called symmetric curvature tensor

$$\chi_{ij} = \frac{1}{2}\left(\theta_{i\|j} + \theta_{j\|i}\right) \tag{3}$$

in which the quantities $\theta^i = \Im^{ijk}u_{k\|j}/2$ represent the components of classical rotation vector. Since $\mathscr{U} = \mathscr{U}(\varepsilon_{ij}, \chi_{ij})$, the following relations hold

$$\sigma^{ij} = \frac{\partial\mathscr{U}}{\partial\varepsilon_{ij}} \quad \mu^{ij} = \frac{\partial\mathscr{U}}{\partial\chi_{ij}} \tag{4}$$

For linearly elastic isotropic materials, expanding the strain energy density in Maclaurin's series results in the following constitutive relations [29].

$$\begin{aligned} \sigma^{ij} &= \lambda\varepsilon^k_k g^{ij} + 2\mu\varepsilon^{ij} \\ \mu^{ij} &= 2\mu\ell^2\chi^{ij} \end{aligned} \tag{5}$$

where λ and μ are Lame constants and ℓ denotes an additional independent material length scale parameter corresponding to rotation gradients.