Composites Part B 86 (2016) 229–242

Contents lists available at ScienceDirect

Composites Part B

journal homepage: www.elsevier.com/locate/compositesb

Modeling of composite plates based on Reissner's Mixed Variational Theorem with variables separation

P. Vidal*, L. Gallimard, O. Polit

LEME — EA 4416, Université Paris Ouest, 50 Rue de Sèvres, 92410, Ville d'Avray, France

ARTICLE INFO

Article history:
Received 10 July 2015
Received in revised form
11 September 2015
Accepted 22 September 2015
Available online 28 October 2015

Keywords:
A. Laminates
B. Elasticity
C. Computational modelling
Separation of variables

ABSTRACT

In this work, the modeling of laminated composite plates is performed through a variables separation approach based on a Reissner's Variational Mixed Theorem (RMVT). Both the displacement and transverse stress fields are approximated as a sum of separated functions of the in-plane coordinates x,y and the transverse coordinate z. This choice yields to a non-linear problem that can be solved by an iterative process. That consists of solving a 2D and 1D problem successively at each iteration. In the thickness direction, a fourth-order expansion in each layer is considered. For the in-plane description, classical Finite Element method is used.

Numerical examples involving several representative laminates are addressed to show the accuracy of the present LayerWise (LW) method. It is shown that it can provide quasi-3D results less costly than classical LW computations. In particular, the estimation of the transverse stresses which are of major importance for damage analysis is very good.

1. Introduction

Composite and sandwich structures are widely used in the weight-sensitive industrial applications due to their excellent mechanical properties, especially their high specific stiffness and strength. In this context, they can be subjected to severe mechanical loading. For composite design, accurate knowledge of displacements and stresses is required. So, it is important to take into account transverse shear deformation due to the low ratio of transverse shear modulus to axial modulus, or failure due to delamination. In fact, they can play an important role on the behavior of structures in services, which leads to evaluate precisely their influence on local stress fields in each layer, particularly at the interface between layers.

Theoretical models for heterogeneous structures can be classified as follows:

- the Equivalent Single Layer Models (ESLM), where the classical Love-Kirchhoff (CLT [1]), Reissner-Mindlin (FSDT [2]) and higher-order models (HSDT [3–7]) can be found for plates. The first one leads to inaccurate results for composites because both transverse and normal strains are neglected. The second one needs a shear correction factor. Moreover, transverse shear and normal stress continuity conditions at the interfaces between layers are violated for all of them.
- the Layer-Wise Models (LWM) that aim at overcoming the restriction of the ESL. The reader can refer to the works of Pagano [8] and Reddy [9,10]. See also [11–13].

According to Reddy [14], the number of unknowns remains independent of the number of constitutive layers in the ESLM, while the same set of variables is used in each layer for the LWM. Note that excellent reviews and extensive assessments have been made in the following articles [15–20].

Nevertheless, in the framework of the failure analysis of composite structures, the prediction of the interlaminar stresses is of major interest. In particular, the difficulty is to well-describe the interlaminar continuous transverse stresses. Most of the ESLM fail, requiring the use of post-processing treatment [21–23]. Another way is based on the introduction of interface conditions into higher-order model pertaining to the ESLM or to the LWM. This permits to reduce the number of unknowns and can be viewed as Zig-Zag models [24–27]. Unfortunately, for very severe cases, some limitations appear (cf. [28]).

* Corresponding author.
E-mail address: philippe.vidal@u-paris10.fr (P. Vidal).

http://dx.doi.org/10.1016/j.compositesb.2015.09.055
1359-8368/

To overcome these drawbacks, alternative formulations to the displacement-based approach have been developed. For that, the hybrid formulation [29] and partial hybrid formulation [30] have been proposed to improve the accuracy of the transverse shear stresses. It has been extended to a larger domain of applications in Ref. [31] with a three-field Hu-Washizu functional principle. An alternative method is based on the weak compatibility condition on the transverse normal strain. It has been carried out with a zig-zag model in Refs. [32–34] ensuring the continuity of the transverse shear stresses, and with HSDT in Ref. [35] also avoiding the use of the transverse normal stress as unknowns. Nevertheless, the transverse stresses can be introduced as unknowns as in Ref. [36] to derive a mixed Layerwise FE model based on the minimum potential energy principle. Recently, the mixed least-square formulation in conjunction with a FSDT model or LW approach has been successfully applied to laminated plates [37,38]. This formulation seems to have good properties in the framework of Finite Element method, but the number of unknowns can be high as displacements, transverse stresses and in-plane deformations have to be computed. It should be also noted that the mixed approach of Pagano [39] has been employed by Thai et al. [40] for formulating LW finite plate elements.

For the present work, the partially Reissner's Mixed Variational Theorem (RMVT) assuming two independent fields for displacement and transverse stress variables is used in conjunction with a variables separation. The resulting approach ensures a priori interlaminar continuous transverse stress fields. The RMVT approach comes from the works of Reissner, see Refs. [41,42]. It was first applied for multilayered structures in Ref. [43] and then, in Ref. [44] with higher order displacement field and [45] with a Layerwise approach for both displacements and transverse stress fields. Afterward, the approach was widely developed with a systematic approach based on the Carrrera's Unified Formulation to provide a large panels of 2D models for composite structures based on ESL and/or LW descriptions of the unknowns [46–48]. For a further discussion, the reader can refer to [49]. Nevertheless, the LW approach with a RMVT formulation drives to high computational cost. Thus, a promising alternative approach consists of the introduction of the separation of variables which could overcome these drawbacks. Interesting features has been shown in the reduction model framework [50]. So, the aim of the present paper is to assess this particular representation of the unknowns in the framework of a mixed formulation to model laminated and sandwich plates. Thus, both displacements and transverse stresses are written under the form of a sum of products of bidimensional polynomials of (x,y) and unidimensional polynomials of z. A piecewise fourth-order Lagrange polynomial of z is chosen. As far as the variation with respect to the in-plane coordinates is concerned, a 2D eight-node quadrilateral FE is employed. Using this method, each unknown function of (x,y) is classically approximated using one degree of freedom (dof) per node of the mesh and the LW unknown functions of z are global for the whole plate. Finally, the deduced non-linear problem implies the resolution of two linear problems alternatively. This process yields to a 2D and a 1D problems in which the number of unknowns is much smaller than a classical Layerwise approach. Note that this type of method has been successfully applied with a displacement-based framework in Refs. [51,52].

We now outline the remainder of this article. First, the RMVT mechanical formulation is described and the separation of the in-plane and out-of-plane stresses is introduced. Then, the principles of the PGD are defined in the framework of our study. The particular assumption on the displacements and the transverse stresses yields to a non-linear problem. An iterative process is chosen to solve this one. The FE discretization is also given. Finally, numerical tests are performed for very thick to thin laminated and sandwich plates. Different stacking sequences are also considered. The behavior of the approach are presented and illustrated. The accuracy of the results is assessed with respect to exact reference solutions [53] and results available in open literature. We also focus on the distributions of the transverse stresses along the thickness which are continuous at the interface between adjacent layers. The results issued from the displacement-based approach with a variables separation [52] are given for further assessments.

2. Reference problem description: the governing equations

Let us consider a plate occupying the domain $\mathscr{V} = \Omega \times \Omega_z$ with $\Omega = [0,a] \times [0,b]$ $\Omega_z = [-h/2,h/2]$ in a Cartesian coordinate (x,y,z). The plate is defined by an arbitrary region Ω in the (x,y) plane, located at the midplane for $z = 0$, and by a constant thickness h. See Fig. 1.

2.1. Constitutive relation

Stresses $\boldsymbol{\sigma}$ and strains $\boldsymbol{\varepsilon}$ are split into two groups:

$$\begin{aligned} \boldsymbol{\sigma}_p^T &= [\sigma_{11}\ \sigma_{22}\ \sigma_{12}], \quad \boldsymbol{\sigma}_n^T = [\sigma_{13}\ \sigma_{23}\ \sigma_{33}], \\ \boldsymbol{\varepsilon}_p^T &= [\varepsilon_{11}\ \varepsilon_{22}\ \gamma_{12}], \quad \boldsymbol{\varepsilon}_n^T = [\gamma_{13}\ \gamma_{23}\ \varepsilon_{33}] \end{aligned} \tag{1}$$

where the subscripts n and p denote transverse and in-plane values, respectively.

The plate can be made of NC perfectly bonded orthotropic layers. Using the separation between transverse and in-plane components, the three dimensional constitutive law of the k^{th} layer is given by:

$$\begin{cases} \boldsymbol{\sigma}_{pH}^{(k)} = \mathbf{Q}_{pp}^{(k)} \boldsymbol{\varepsilon}_{pG} + \mathbf{Q}_{pn}^{(k)} \boldsymbol{\varepsilon}_{nG} \\ \boldsymbol{\sigma}_{nH}^{(k)} = \mathbf{Q}_{np}^{(k)} \boldsymbol{\varepsilon}_{pG} + \mathbf{Q}_{nn}^{(k)} \boldsymbol{\varepsilon}_{nG} \end{cases} \tag{2}$$

where

$$\mathbf{Q}_{pp}^{(k)} = \begin{bmatrix} Q_{11}^{(k)} & Q_{12}^{(k)} & Q_{16}^{(k)} \\ Q_{12}^{(k)} & Q_{22}^{(k)} & Q_{26}^{(k)} \\ Q_{16}^{(k)} & Q_{26}^{(k)} & Q_{66}^{(k)} \end{bmatrix}$$
$$\mathbf{Q}_{pn}^{(k)} = \mathbf{Q}_{np}^{(k)T} = \begin{bmatrix} 0 & 0 & Q_{13}^{(k)} \\ 0 & 0 & Q_{23}^{(k)} \\ 0 & 0 & Q_{36}^{(k)} \end{bmatrix} \quad \mathbf{Q}_{nn}^{(k)} = \begin{bmatrix} Q_{55}^{(k)} & Q_{45}^{(k)} & 0 \\ Q_{45}^{(k)} & Q_{44}^{(k)} & 0 \\ 0 & 0 & Q_{33}^{(k)} \end{bmatrix} \tag{3}$$

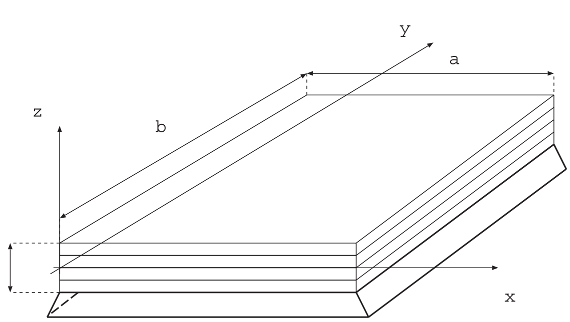

Fig. 1. The laminated plate and coordinate system.