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Probabilistic multiconstraints optimization of cooling channels in ceramic matrix composites

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ABSTRACT

This paper presents a computational reliable optimization approach for internal cooling channels in Ceramic Matrix Composite (CMC) under thermal and mechanical loadings. The algorithm finds the optimal cooling capacity of all channels (which directly minimizes the amount of coolant needed). In the first step, available uncertainties in the constituent material properties, the applied mechanical load, the heat flux and the heat convection coefficient are considered. Using the Reliability Based Design Optimization (RBDO) approach, the probabilistic constraints ensure the failure due to excessive temperature and deflection will not happen. The deterministic constraints restrict the capacity of any arbitrary cooling channel between two extreme limits. A "series system" reliability concept is adopted as a union of mechanical and thermal failure subsets. Having the results of the first step for CMC with uniformly distributed carbon (C-) fibers, the algorithm presents the optimal layout for distribution of the C-fibers inside the ceramic matrix in order to enhance the target reliability of the component. A sequential approach and B-spline finite elements have overcome the cumbersome computational burden. Numerical results demonstrate that if the mechanical loading dominates the thermal loading, C-fibers distribution can play a considerable role towards increasing the reliability of the design.

1. Introduction

The main disadvantage of monolithic ceramics is their low fracture toughness. Thus, carbon fibers are added to increase their damage tolerance while maintaining other advantages (for instance lower density and higher maximum operating temperature compared to metals or high erosion and corrosion resistance).

As a common reinforcing ingredient, C-fibers degrade in an oxidizing atmosphere beyond 450 °C [1]. Although multilayer protection coatings hinder degradation to a degree, the coating process may itself result in formation of interphasial cracks. Preventing high temperature zones in the component might be a better solution. Such a solution however calls for a multidisciplinary approach accounting for material selection, coating and internal cooling design.

This paper presents a computational framework for an efficient and reliable internal cooling network for a typical component made of CMC. Although some attempts to optimize internal cooling system of a monotonic metallic turbine blade exist, the currently known approaches are limited to using heuristic optimization methods, particularly Genetic Algorithm (GA) which is computationally expensive. For example, Dennis et al. [2] used parallel genetic algorithm to optimize locations and discrete radii of a large number of small circular cross-section coolant passages. Nagaiah and Geiger [3] used NSGA-II as a multiobjective evolutionary algorithm optimizing the rib design inside a 2D cooling channel of a gas turbine blade. In both works an external commercial finite element package is used for the thermal analysis.

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Regardless of the optimization technique, another major drawback of current methods is their deterministic nature. Actual characteristics of a composite material (including CMC) involve many uncertainties. These emanate from a variety of sources such as constituent material properties, manufacturing and process imperfections, loading conditions and geometry (a classification is presented in Ref. [4]). Neglecting the role of uncertainties in composite materials might result in either unsafe or unnecessary conservative design. This research focuses not only on optimal but also on reliable design of a typical internal cooling network within a CMC using a non-heuristic method and accounting for uncertainties.

We take advantage of sequential optimization approach [5] and propose a two stage optimization process in which the stages are sequentially linked to each other. In the first stage, it is assumed that C-fibers are uniformly distributed in the ceramic matrix. Then by using RBDO, the outputs of the first stage which are optimal capacities of the cooling channels, are exported into the next stage. In the second stage, the optimizer takes these inputs and uses the adjoint sensitivity technique adopted for the coupled elastic and thermal fields and eventually provides an optimal distribution of the C-fibers within the design domain in order to enhance the target reliability of the component.

The remainder of this paper is organized as follows: In Section 2 and Section 3, the thermoelastic finite element formulations and structural reliability concept are briefly discussed. The optimization methodology is explained in Section 4. Afterwards, case studies in Section 5 and concluding remarks in Section 6 are presented.

2. Thermoelastic formulation

The steady-state governing equation and boundary conditions for a temperature field in a 2D isotropic solid with domain Ω and boundary Γ are [6].

$$\left(k_{ij}\theta_{,j}\right)_{,i} + Q = 0 \quad \text{in } \Omega \tag{1}$$

$$\theta = \theta_\Gamma \quad \text{on } \Gamma_1 \text{ Essential boundary} \tag{1.a}$$

$$-n_i k_{ij}\theta_{,j} = q_\Gamma \quad \text{on } \Gamma_2 \text{ Heat flux boundary} \tag{1.b}$$

$$-n_i k_{ij}\theta_{,j} = h(\theta - \theta_\infty) \quad \text{on } \Gamma_3 \text{ Convection boundary} \tag{1.c}$$

$$-n_i k_{ij}\theta_{,j} = 0 \quad \text{on } \Gamma_4 \text{ Adiabatic boundary} \tag{1.d}$$

where k_{ij}, Q, and θ denote the thermal conductivity, internal uniform heat source and temperature field, respectively; n_i is component of the unit outward normal to the boundary, h is the heat convection coefficient, q_Γ is the prescribed heat flux and θ_∞ is the temperature of the surrounding medium in convection process.

The governing equation and boundary conditions for a linear elastic solid are given by

$$\sigma_{ij,j} + b_i = 0 \quad \text{in } \Omega \tag{2}$$

$$u_i = u_\Gamma \quad \text{on } \Gamma_u \text{ Essential boundary} \tag{2.a}$$

$$\sigma_{ij}n_i = t_\Gamma \quad \text{on } \Gamma_t \text{ Natural boundary} \tag{2.b}$$

where σ and b denote the stress and body force. u_Γ and t_Γ are the given displacement and traction on the essential and natural boundaries, respectively.

The heat and elastic problems are linked by the following stress, strain and thermal expansion relation

$$\sigma_{ij} = \delta_{ij}\lambda_L\varepsilon_{kk} + 2\mu_L\varepsilon_{ij} - \delta_{ij}(3\lambda_L + 2\mu_L)\alpha\Delta\theta \tag{3}$$

where λ_L and μ_L are Lamé's constants, α is the thermal expansion coefficient and $\Delta\theta$ is the temperature change with respect to the reference temperature which is assumed zero here.

A weighted residual weak form of the boundary value problem (Eq. 1-1.d) can be written as a generalized functional I

$$I(\theta) = \int_\Omega w\left[\left(k_{ij}\theta_{,j}\right)_{,i} + Q\right] d\Omega \tag{4}$$

where w denotes the sufficiently differentiable test function. The functional $I(\theta)$ can be written as

$$I(\theta) = \int_\Omega \frac{1}{2}\left[k_{x_1}\left(\frac{\partial\theta}{\partial x_1}\right)^2 + k_{x_2}\left(\frac{\partial\theta}{\partial x_2}\right)^2\right] d\Omega - \int_\Omega \theta Q d\Omega + \int_{\Gamma_2} \theta q_\Gamma d\Gamma + \int_{\Gamma_3} h\theta\left(\frac{1}{2}\theta - \theta_\infty\right) d\Gamma \tag{5}$$

considering δ as the variational operator, the Bubnov–Galerkin weak form for the heat transfer problem can be obtained as follows

$$\int_\Omega \delta(\nabla\theta)^T \boldsymbol{K}_c \nabla\theta d\Omega - \int_\Omega \delta\theta^T Q d\Omega + \int_{\Gamma_2} \delta\theta^T q_\Gamma d\Gamma + \int_{\Gamma_3} \delta\theta^T h\theta d\Gamma - \int_{\Gamma_3} \delta\theta^T h\theta_\infty d\Gamma = 0 \tag{6}$$

The strains arising from boundary loadings and body forces induce only small temperature changes which can be ignored in the analysis. Thus, the semi-coupled theory of thermoelasticity is employed here. The heat governing equations are firstly solved to obtain the temperature field. Then, the body forces induced by the temperature field are used along with the other applied forces to calculate the final response of the elastic body. Using the Bubnov–Galerkin weak form

$$\int_\Omega \delta(\boldsymbol{\varepsilon}(\boldsymbol{u}) - \boldsymbol{\varepsilon}_0(\boldsymbol{u}))^T \boldsymbol{C}(\boldsymbol{\varepsilon}(\boldsymbol{u}) - \boldsymbol{\varepsilon}_0(\boldsymbol{u})) d\Omega - \int_{\Gamma_t} \delta\boldsymbol{u}^T t_\Gamma d\Gamma - \int_\Omega \delta\boldsymbol{u}^T \boldsymbol{b} d\Omega = 0 \tag{7}$$

In this work quadratic B-spline basis functions (similar to our previous works [15–21]) are selected as the test function w. They are also employed to approximate the displacement and temperature fields

$$\boldsymbol{u}(x,y) = \sum_{i=1}^{n}\sum_{j=1}^{m} N_{i,j}^{p,q}(\xi,\eta)\boldsymbol{u}_{i,j} = \boldsymbol{N}\boldsymbol{u} \tag{8.a}$$

$$\boldsymbol{\theta}(x,y) = \sum_{i=1}^{n}\sum_{j=1}^{m} N_{i,j}^{p,q}(\xi,\eta)\boldsymbol{\theta}_{i,j} = \boldsymbol{N}\boldsymbol{\theta} \tag{8.b}$$