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Simulation of a cascaded longitudinal space charge amplifier for coherent radiation generation

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ABSTRACT

Longitudinal space charge (LSC) effects are generally considered as harmful in free-electron lasers as they can seed unfavorable energy modulations that can result in density modulations with associated emittance dilution. This "micro-bunching instabilities" is naturally broadband and could possibly support the generation of coherent radiation over a broad region of the spectrum. Therefore there has been an increasing interest in devising accelerator beam lines capable of controlling LSC induced density modulations. In the present paper we refine these previous investigations by combining a grid-less space charge algorithm with the popular particle-tracking program ELEGANT. This high-fidelity model of the space charge is used to benchmark conventional LSC models. We finally employ the developed model to investigate the performance of a cascaded LSC amplifier using beam parameters comparable to the ones achievable at Fermilab Accelerator Science & Technology (FAST) facility currently under commissioning at Fermilab.

1. Introduction

Longitudinal-space-charge-driven micro-bunching instabilities arising in bunch compressors were predicted and observed over the last decade [1–3]. It was recently proposed to employ such micro-bunching instability mechanism to form attosecond structures on the bunch current distribution for the subsequent generation of coherent radiation pulses [4].

A possible beam line configuration capable of enabling the micro-bunching instability is relatively simple. It essentially consists of focusing section (e.g. FODO cells) where energy modulations due to the LSC impedance accumulate, followed by a longitudinally dispersive section. The latter section, by introducing an energy dependent path length, converts the incoming energy modulation into a density modulation. Such an elementary cell is often referred to as a LSC amplifier (LSCA). Most of the beamlines studied so far consider a longitudinally dispersive section arranged as a bunch compression chicane [or bunch compressor (BC)]; see Fig. 1. Several of these LSCA modules are concatenated so to result in a large final density modulation. We further assume the compression process in the chicane is linear [the incoming longitudinal phase space (LPS) does not have any nonlinear correlations]. Such a modulated beam, when participating in a radiation-generation process, can produce coherent radiation at wavelengths comparable to the spectral range of the final density modulations.

The purpose of this paper is two-fold. The paper first introduces a fully three dimensional (3D) multi-scale space-charge algorithm adapted from Astrophysics [5]. The algorithm is used to discuss some limitations of the one-dimensional LSC impedance model commonly employed in LSCA investigations. Using the latter benchmarked algorithm, we then investigate a possible LSCA beamline configuration similar to the one studied in [4]. Finally, we estimate the generation of undulator radiation seeded by the LCSA. In contrast to Ref. [4] our study considers the case of a ~ 500 A 300-MeV electron beam produced in a conventional superconducting linac.

2. Mechanism for longitudinal space charge amplifiers

Charged-particle beams are subject to self-interaction via velocity and radiation fields. In absence of radiation processes (i.e. acceleration), the effect of velocity fields (i.e. space charge) dominates and its regime varies with the bunch density. Under a simple 1D approximation, a comparison of the Debye length λ_D to the root-mean-squared (rms) transverse beam size $\sigma_\perp$ and mean inter-particle distance $\Lambda_p \simeq n_e^{-1/3}$ (where n_e is the electronic density) provides a criterion to assess the importance of space charge effects on the beam dynamics. When $\sigma_\perp < \lambda_D$ space charge

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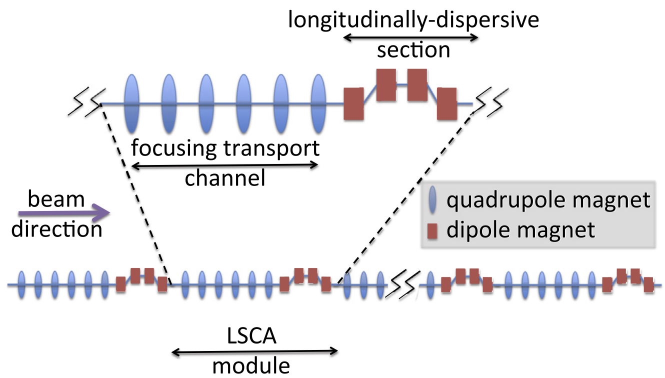

Fig. 1. Overview of a cascaded longitudinal-space-charge amplifier (LSCA) composed of several LSCA modules. Each LSCA module incorporate a focusing channel and a longitudinally dispersive section. The (red) rectangles and (blue) ellipses respectively represent dipole and quadrupole magnets.

effects are significant and often computed using the mean-field approximation (i.e. the space charge force is derived from the electrostatic potential associated to the particle distribution) commonly implemented in particle-in-cell (PIC) algorithms. However, when $\lambda_D \sim \mathcal{O}(\Lambda_p)$, particle-to-particle "binary" interactions play an important role and are needed to be accounted for [6].

As the beam is accelerated the transverse and longitudinal space-charge forces reduce respectively as $\mathcal{O}(1/\gamma^2)$ and $\mathcal{O}(1/\gamma^3)$ where γ is the Lorentz factor. At the macroscopic level, e.g. for spatial scale comparable to the bunch sizes, the space charge can be accurately described by a mean field approach [7]. However, in high-brightness beams – beams with low fractional momentum spread – the weakened longitudinal-space charge (LSC) force can still influence the beam dynamics at a microscopic level – i.e. for spatial scales smaller than the bunch sizes – and small density modulations (e.g. due to noise or imperfections) can result in LCS-driven energy modulations. In this latter regime, the LSC is generally treated with a one-dimensional (1D) model.

To illustrate the main results of the 1-D model, we consider a simple beam line consisting of a drift with length L_d (where the beam is transversely contained) followed by a chicane with longitudinal dispersion R_{56}. It is customary to characterize the strength of the micro-bunching instability by associating the density gain defined as

$$G(k) = \frac{b_i(k)}{b_f(k)} \tag{1}$$

where $k \equiv \frac{2\pi}{\lambda}$ and λ is the observation wavelength and $b_{i,f}$ are respectively the initial and final bunching factors defined as

$$b(\omega) = \frac{1}{N}\left|\sum_n \exp(-i\omega t_n)\right| \tag{2}$$

where t_n is the temporal coordinate of the n-th macroparticle, N is the total number of particles and $\omega \equiv kc$. In the latter equation we assume the beam's longitudinal density to follow the Klimontovich distribution $\rho(t) = \frac{1}{N}\sum_{j=1}^{N}\delta(t-t_j)$.

The gain for this simple beam line can be shown to be proportional to the impedance $Z(k,r)$ [8] following

$$G = Ck|R_{56}|\frac{I}{\gamma I_A}\frac{4\pi L_d|Z(k,r)|}{Z_0}e^{-\frac{1}{2}C^2k^2R_{56}^2\sigma_\delta^2} \tag{3}$$

where I_A=17 kA is the Alfvèn current, σ_δ is the rms fractional energy spread, $C \equiv \langle z\delta\rangle/\sigma_z$ is the chirp, and $Z_0 \equiv 120\pi$ is the free-space impedance.

The exponential term in Eq. (3) induces a high-frequency cut-off of the modulation

$$R_{56} \approx -\frac{c}{\omega\sigma_\delta}. \tag{4}$$

Note, that after traveling through a BC, the modulation wavelength will be shortened by a compression factor $\kappa \equiv (1+R_{56}C)$. Although the impedance $Z(k,r)$ is partially determined by the properties of the wakefields inside the BC [8], the LSC has much stronger effect in amplifying density modulations [4,9].

For a transversely Gaussian cylindrically symmetric beam the LSC impedance is given by [10]

$$Z(k) = -i\frac{Z_0}{\pi\gamma\sigma_\perp}\frac{\xi_{\sigma_\perp}}{4}e^{\xi_{\sigma_\perp}^2/2}\,\mathrm{Ei}\left(-\frac{\xi_{\sigma_\perp}^2}{2}\right) \tag{5}$$

where $Z_0 = 120\pi$ is the free-space impedance, $\mathrm{Ei}(x) \equiv -\int_{-x}^{\infty} dt\, e^{-t}/t$, $\sigma_\perp$ is the rms beam size and $\xi_{\sigma_\perp} \equiv k\sigma_\perp/\gamma$. Similar expression for a transversely uniform beam is provided in [11].

The maximum of Eq. (5) is achieved at $\xi_{\sigma_\perp} \approx 1$, therefore the optimal wavelength of the density modulation will be located around

$$\lambda_{opt} = 2\pi\sigma_\perp/\gamma. \tag{6}$$

3. Simulation procedure and benchmarking

The nature of space charge forces lies in particle-to-particle Coulomb interaction. Direct summation of the forces yields to $\mathcal{O}(N^2)$ growth where N is the number of macroparticles, which makes it impossible to compute at large N. Several approximation techniques can be used: mean-field on a grid approximation [12], one-dimensional space charge impedance [10], analytical sub-beams or ensembles model [13], rigid-slice approximation [7]. All of those methods reduce the problem's complexity via some approximations which ultimately limits the maximum attainable spatial resolution. Most recent attempt used a three-dimensional-grid space charge algorithm based on a periodic boundary [4].

From another point of view, space charge problem is very similar to the well-known N-body problem in celestial mechanics. One of the most effective algorithms for the gravitational N-body problem is the so-called "tree" or Barnes–Hut (BH) algorithm [5], which scales as $\mathcal{O}(N \log N)$. In this paper we present the results obtained using a modified version of the code available in [14]. Such approach was successfully employed to simulate early-stage beam dynamics in photocathodes [15] and laser ion cooling [16].

In brief, the BH algorithm initially surrounds the bunch distribution in a cubic cell called a root cell. The root cell is divided into 8 sub-cells recursively, until it reaches the point where a single sub-cell contains just one particle. Then forces only between nearby cells are calculated directly, and the cells far away from each other are treated as two large macroparticles with the total charge placed in the cell's center of mass. The process of calculating net forces starts from the root cell and recursively parses the cell hierarchy until it reaches the size of the smallest cell that is predefined as a precision parameter. Thus, the algorithm is significantly faster than a direct summation method. The BH method does not preserve full Hamiltonian, yet for relatively small precision parameter the difference between direct summation is comparably small [5]. It should be pointed out in the direct summation part (for neighboring cells) the BH algorithm also implements a local smoothing of the potential to avoid singularities [5].

Another more efficient fast multipole method (FMM) algorithm has been recently developed [17,18] and will be eventually used in further refinement of our work. Though FMM algorithms are more