Composites Part B 81 (2015) 231–250

Contents lists available at ScienceDirect

Composites Part B

journal homepage: www.elsevier.com/locate/compositesb

Numerical and exact models for free vibration analysis of cylindrical and spherical shell panels

F. Tornabene [a], S. Brischetto [b,*], N. Fantuzzi [a], E. Viola [a]

[a] DICAM Department, University of Bologna, Italy
[b] Department of Mechanical and Aerospace Engineering, Politecnico di Torino, Corso Duca degli Abruzzi, 24, 10129 Torino, Italy

ARTICLE INFO

Article history:
Received 20 May 2015
Received in revised form
16 July 2015
Accepted 27 July 2015
Available online 5 August 2015

Keywords:
C. Analytical modelling
C. Computational modelling
C. Finite element analysis (FEA)
C. Numerical analysis

ABSTRACT

The present paper shows a comparison between classical two-dimensional (2D) and three-dimensional (3D) finite elements (FEs), classical and refined 2D generalized differential quadrature (GDQ) methods and an exact three-dimensional solution. A free vibration analysis of one-layered and multilayered isotropic, composite and sandwich cylindrical and spherical shell panels is made. Low and high order frequencies are analyzed for thick and thin simply supported structures. Vibration modes are investigated to make a comparison between results obtained via the FE and GDQ methods (numerical solutions) and those obtained by means of the exact three-dimensional solution. The 3D exact solution is based on the differential equations of equilibrium written in general orthogonal curvilinear coordinates. This exact method is based on a layer-wise approach, the continuity of displacements and transverse shear/normal stresses is imposed at the interfaces between the layers of the structure. The geometry for shells is considered without any simplifications. The 3D and 2D finite element results are obtained by means of a well-known commercial FE code. Classical and refined 2D GDQ models are based on a generalized unified approach which considers both equivalent single layer and layer-wise theories. The differences between 2D and 3D FE solutions, classical and refined 2D GDQ models and 3D exact solutions depend on several parameters. These include the considered mode, the order of frequency, the thickness ratio of the structure, the geometry, the embedded material and the lamination sequence.

1. Introduction

The present work proposes a free vibration analysis of simply-supported one-layered and multilayered isotropic, composite and sandwich cylindrical and spherical shell panels. Low and high frequencies and related modes are investigated. The importance of this topic has been extensively discussed in the reports by Leissa [1,2], in the book by Werner [3] and in the work by Brischetto and Carrera [4], among others.

The main aim of this work is the comparison between the results obtained by means of an exact three-dimensional (3D) solution, and those obtained by means of the classical two-dimensional (2D) and three-dimensional (3D) finite element methods (FEM) and by means of the classical and refined 2D generalized differential quadrature (GDQ) models. The proposed exact 3D solution was developed by Brischetto in Refs. [5–10] where the differential equations of equilibrium written in general orthogonal curvilinear coordinates were exactly solved by means of the exponential matrix method. The 2D and 3D FE results were obtained by means of the commercial finite element code Straus7 [11]. In this study, Straus7 was chosen for its user interface simplicity. In particular, it is easy to control the orientation of the plies in the stacking sequences and the location of the boundary conditions. The GDQ models were developed from the geometric description of the middle surface of shells. This description was carried out analytically using differential geometry [12,13]. The GDQ method can discretize any partial or total derivative using a weighted linear sum and the functional values in the definition domain. Therefore, it is easy and straightforward to transform the analytic description of a structure into a discrete set of equations [14–29]. The GDQ method was introduced by Shu and Richards [30] to solve physics problems related to fluids. A lot of advances have been made to date, which are summarized in the review papers [31–33]. It has been demonstrated that the GDQ method is a flexible and easy procedure for solving systems of partial differential equations in their strong form. Further applications related to finite elements in

* Corresponding author. Tel.: +39 011 090 6813; fax: +39 011 090 6899.
E-mail address: salvatore.brischetto@polito.it (S. Brischetto).

http://dx.doi.org/10.1016/j.compositesb.2015.07.015
1359-8368/

strong form using the GDQ method can be found in Refs. [34–40] in the case of vibrations of arbitrarily shaped structures.

In the most general case of exact three-dimensional analyses, the number of frequencies for a free vibration problem is infinite: three displacement components (3 degrees of freedom DOF) in each point (points are ∞ in the 3 directions x, y, z) leads to $3 \times \infty^3$ vibration modes. Assumptions are made in the thickness direction z in the case of a 2D plate/shell model, the three displacements in each point are expressed in terms of a given number of degrees of freedom (NDOF) through the thickness direction z. NDOF varies from theory to theory. As a result, the number of vibration modes is $NDOF \times \infty^2$ in the case of exact 2D models. For exact 1D beam models, the number of vibration modes is $NDOF \times \infty^1$. In the case of 2D computational models, such as the Finite Element (FE) method or the generalized differential quadrature (GDQ) models, the number of modes is a finite number. This number coincides with the total number of employed degrees of freedom: $\sum_1^{Node} NDOF_i$, where *Node* denotes the number of nodes used in the FE mathematical model or in the GDQ analysis, and $NDOF_i$ is the NDOF through the thickness direction z in the i-node. It is clear that some modes are not calculated by simplified models (such as computational two-dimensional models) [4]. In order to make a comparison between the 2D and 3D FE free vibration results, the 2D GDQ results, and the 3D exact free vibration results, the investigation of the vibration modes is mandatory in order to understand which frequencies must be compared.

The most relevant papers concerning 3D solutions for free vibration analysis of shell structures are shown below. The coupled free vibrations of a transversely isotropic cylindrical shell embedded in an elastic medium were analyzed in Ref. [41] where the three-dimensional elastic solution used three displacement functions. Free vibrations of simply-supported cylindrical shells were studied in Ref. [42] on the basis of three-dimensional exact theory. Extensive frequency parameters were obtained by solving frequency equations. The free vibrations of simply-supported cross-ply cylindrical and doubly-curved laminates were investigated in Ref. [43]. The three-dimensional equations of motion were reduced to a system of coupled ordinary differential equations and then solved using the power series method. The three-dimensional free vibrations of a homogenous isotropic, viscothermoelastic hollow sphere were studied in Ref. [44]. The surfaces were subjected to stress-free, thermally insulated or isothermal boundary conditions. The exact three-dimensional vibration analysis of a trans-radially isotropic, thermoelastic solid sphere was analyzed in Ref. [45]. The governing partial differential equations in Refs. [44,45] were transformed into a coupled system of ordinary differential equations. The Fröbenious matrix method was employed to obtain the solution. Soldatos and Ye [46] proposed exact, three-dimensional, free vibration analysis of angle-ply laminated thick cylinders with a regular symmetric or a regular antisymmetric angle-ply lay-up. Armenakas et al. [47] proposed a self-contained treatment of the problem of plane harmonic wave propagation along a hollow circular cylinder in the framework of the three-dimensional theory of elasticity. A comparison between a refined two-dimensional analysis, a shear deformation theory, the Flügge theory and an exact elasticity analysis was proposed in Ref. [48] for frequency investigation. Further details about the Flügge classical thin shell theory concerning the free vibrations of cylindrical shells with elastic boundary conditions can be found in Ref. [49]. Other comparisons between two-dimensional closed form solutions and exact 3D elastic analytical solutions for the free vibration analysis of simply supported and clamped homogenous isotropic circular cylindrical shells were also proposed in Ref. [50]. Vel [51] extended exact elasticity solutions to functionally graded cylindrical shells. The three-dimensional linear elastodynamic equations were solved using suitable displacement functions that identically satisfy the boundary conditions. Loy and Lam [52] obtained the governing equations using an energy minimization principle. A layer-wise approach was proposed to study the vibration of thick circular cylindrical shells on the basis of the three-dimensional theory of elasticity. Wang et al. [53] proposed a three-dimensional free vibration analysis of magneto-electro-elastic cylindrical panels. Further results about the three-dimensional analysis of shells, where the solutions are not given in closed form, can be found in Ref. [54] for the dynamic stiffness matrix method and in Refs. [55,56] for the three-dimensional Ritz method for the vibration of spherical shells.

Conical shells were parametrically investigated using the GDQ method in the works [57–59]. Free vibrations of spherical and other revolution shells were proposed in Refs. [25–29,60–66]. Doubly-curved composite shells including new physical effects were proposed in the papers [14–22]. Each of the previous citations was fundamental for the present GDQ model because the code had been previously tested for different cases.

The three-dimensional analyses proposed in the literature show free vibrations of plates or shells. They separately analyze shell or plate geometries and they do not give a general overview for both structures. The proposed exact 3D model uses a general formulation for several geometries (square and rectangular plates, cylindrical and spherical shell panels, and cylindrical closed shells). The equations of motion for the dynamic case are written in general orthogonal curvilinear coordinates using an exact geometry for multilayered shells. The system of second order differential equations is reduced to a system of first order differential equations, and subsequently solved exactly using the exponential matrix method and the Navier-type solution. The approach is developed in a layer-wise form imposing the continuity of displacements and transverse shear/normal stresses at each interface. The exponential matrix method was already used in Ref. [67] for the three-dimensional analysis of plates in rectilinear orthogonal coordinates and in Ref. [46] for an exact, three-dimensional, free vibration analysis of angle-ply laminated cylinders in cylindrical coordinates. The equations of motion written in orthogonal curvilinear coordinates are a general form of the equations of motion written in rectilinear orthogonal coordinates in Ref. [67] and in cylindrical coordinates in Ref. [46]. The present 3D equations allow general exact solutions for multilayered plate and shell geometries as already seen in the second author's works [5–10]. In the literature review proposed in this introduction, only a few works analyzed higher order frequencies. Moreover, papers that discuss the comparison between numerical 2D models and exact 3D models are even less frequent. The present work aims to fill this gap, it compares the free frequencies for cylindrical and spherical shell panels obtained by means of the commercial FE code Straus7, the 2D GDQ models, and the exact 3D solution. The proposed 3D exact solution gives results for plates, cylindrical and spherical shell panels, and cylindrical closed shells. However, the comparison with the commercial FE code and the GDQ models is proposed only for cylindrical and spherical shell panels because the plate and cylinder cases were already investigated in Ref. [8]. The aim of the present paper is to understand how to compare these three different methods (exact 3D and numerical FE and GDQ solutions) and also to show the limitations of classical 2D theories.

2. Exact three-dimensional model

The equations of equilibrium written for the case of free vibration analysis of multilayered spherical shells embedding N_L layers with constant radii of curvature R_α and R_β are (the general form for variable radii of curvature can be found in Refs. [68,12])