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Developing a cosmic ray muon sampling capability for muon tomography and monitoring applications

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ABSTRACT

In this study, a cosmic ray muon sampling capability using a phenomenological model that captures the main characteristics of the experimentally measured spectrum coupled with a set of statistical algorithms is developed. The "muon generator" produces muons with zenith angles in the range 0–90° and energies in the range 1–100 GeV and is suitable for Monte Carlo simulations with emphasis on muon tomographic and monitoring applications. The muon energy distribution is described by the Smith and Duller (1959) [35] phenomenological model. Statistical algorithms are then employed for generating random samples. The inverse transform provides a means to generate samples from the muon angular distribution, whereas the Acceptance–Rejection and Metropolis–Hastings algorithms are employed to provide the energy component. The predictions for muon energies 1–60 GeV and zenith angles 0–90° are validated with a series of actual spectrum measurements and with estimates from the software library CRY. The results confirm the validity of the phenomenological model and the applicability of the statistical algorithms to generate polyenergetic–polydirectional muons. The response of the algorithms and the impact of critical parameters on computation time and computed results were investigated. Final output from the proposed "muon generator" is a look-up table that contains the sampled muon angles and energies and can be easily integrated into Monte Carlo particle simulation codes such as Geant4 and MCNP.

1. Introduction

The applicability of cosmic muons for a number of monitoring and imaging applications has been investigated over the years and includes a large number of applications ranging from archeology to volcano imaging, material identification and medical diagnosis [1–8]. It is worth noting the pioneering work of George [10] who used a similar method to infer rock depth covering underground tunnels and that of Alvarez et al. [9] who measured the cosmic ray muon flux attenuation to determine the presence and location of hidden chambers within the Egyptian pyramids. Applications of cosmic ray muons have been proposed for medical examination of comatose patients towards bone density monitoring and determination of the molten nuclear fuel location in nuclear reactors having suffered from the effects of a severe accident similar to the one happened in Chernobyl and Fukushima [6,11].

The role of cosmic ray muons becomes especially important since their use is extended to non-destructive assessment of nuclear materials, such as fuel pellets, fuel rods and fuel assemblies stored within sealed dense containers [12]. Energetic muons have the unique ability to penetrate high density materials allowing the distribution of the material within the object to be inferred from muon measurements [3]. The subsequent scattering and transmission of muons can provide a measurable signal about the structural and chemical composition of the stored materials [13]. Cosmic ray muon tomography is a potential next generation technology in non-destructive evaluation and muons can play a central role encompassing the experience accumulated from recent findings in proton radiography beyond conventional X-ray and gamma-ray techniques.

Successful application of muon technology necessitates the development of classification and imaging algorithms. Algorithm validation and testing is supported by large scale high fidelity Monte Carlo simulations. Simulations of muon–matter interactions are commonly performed using Monte Carlo numerical codes such as Geant4 [14]. Customized Monte Carlo simulations are necessary for examination of the asymptotic behavior of signal processing and classification algorithms [15]. In all cases, accurate and efficient application of Monte Carlo simulations requires knowledge of the muon energy and angular distribution and the ability to generate repeatedly random samples from these distributions [6]. Muons produced in the atmosphere arrive on ground level with a wide range of energies and zenith angles best described by a non-standard differential spectrum. It is therefore

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Table 1
Compilation of muon spectrum experimental measurements.

No.	Authors	Year	Altitude (m)	Momentum range (GeV/c)	Zenith angle range (deg)	Nominal zenith angle (deg)
1	Baber et al. [24]	1968	52	3–1000	0	0
2	Allkofer et al. [25]	1971	10	0.2–1000	0	0
3	Allkofer and Dau [26]	1972	10	0.3–26	0	0
4	Nandi et al. [20]	1972	70	5–1200	0–0.3	0
5	Ayre et al. [27]	1975	70	20–500	0–0.08	0
6	Baxendale et al. [28]	1975	70	7–500	0	0
7	Kellogg et al. [29]	1978	10	50–1700	25.9–34.1	30
				50–1700	70.9–79.1	75
8	Jokisch et al. [23]	1979	10	1–1000	68–82	75
9	Rastin [30]	1984	52	4–3000	0	0
10	Mitsui et al. [31]	1984	10	100–20,000	86–90	89
11	De Pascale et al. [32]	1993	600	0.2–100	0	0
12	Tsuji et al. [21]	1998	5	1.5–250	0–1	0
				2–250	26–34	30
				3–250	59–61	60
				3–250	69–81	75
				3–150	79–81	80
13	Kremer et al. [33]	1999	360	0.2–120	0	0
			1270	0.2–120	0	0
14	Haino et al. [22]	2004	30	0.6–400	0	0
15	L3 collaboration [34]	2005	450	20–3000	0–58	

desirable to develop the capability to generate muons having momentum and zenith angles distributed according to the actual experimentally measured spectrum.

In this paper, a cosmic ray muon sampling capability for use in Monte Carlo simulations is developed. The "muon generator" produces muons with zenith angles in the range 0–90° and energies in the range 1–100 GeV particularly suited for Monte Carlo simulations with emphasis on muon tomographic and monitoring applications. The muon energy distribution is described by a phenomenological model that captures the main characteristics of the experimentally measured spectrum. Statistical algorithms are then employed for generating random samples. The inverse transform provides a means to generate samples from the muon angular distribution, whereas the Acceptance–Rejection and Metropolis–Hastings algorithms are used to provide the energy component. The predictions are subsequently compared with actual spectrum measurements and with estimates from the software library CRY [16]. The main objective is: (a) to develop a muon sampling capability using a phenomenological model coupled with a set of statistical algorithms, (b) to perform benchmark analysis comparing the developed algorithms with experimental measurements and, (c) to examine their response and identify the impact of critical parameters on the computation time and computed results. Main advantages of the "muon generator" include its ability to readily produce any number of muons for any altitude and any combination of zenith angles and energies, along with the fact that it is based on a strong phenomenological model and validated statistical algorithms. A version of the "muon generator" developed for easy and fast integration to Geant4 is available online[1] [17].

A brief description of the muon spectrum characteristics and available experimental measurements is given in Section 2. Sections 3 and 4 provide details related to the phenomenological model and the statistical algorithms developed for generating random samples, respectively. Finally, the results obtained along with the conclusions drawn are presented and discussed in Section 5.

2. Muon spectrum

Cosmic radiation, originating mainly from outside our solar system constantly bombards the upper layers of the Earth's atmosphere [18]. The high energy cosmic rays that enter Earth's atmosphere generate a cascade of secondary rays and relativistic particles. As a result of the processes occurring in the atmosphere, the hadronic component, the electromagnetic component and the muonic component represent the spectrum of particles reaching the surface. Through absorption and decay processes the subatomic particles pions and kaons give rise to a considerable flux of muons that reaches sea level [19]. Cosmic ray muons are charged particles, having approximately 200 times the mass of electron, generated naturally in the atmosphere, and rain down upon the earth at an approximate rate of 10,000 particles $m^{-2}\,min^{-1}$.

Main properties characterizing muon spectrum are integral and differential intensity [19]. Intensity is defined as the number of particles per unit area dA, per unit time dt and per unit solid angle $d\Omega$:

$$I = \frac{dN}{dA dt d\Omega} (\mathrm{cm}^{-2}\ \mathrm{s}^{-1}\ \mathrm{sr}^{-1}) \tag{1}$$

Differential intensity is a useful quantity to describe cosmic radiation and present experimental measurements, defined as the number of particles per unit area dA, per unit time dt and per unit solid angle $d\Omega$ and unit energy dE:

$$D = \frac{dN}{dA dt d\Omega\, dE} (\mathrm{cm}^{-2}\ \mathrm{s}^{-1}\ \mathrm{sr}^{-1}\ \mathrm{GeV}^{-1}) \tag{2}$$

Differential intensity is a function of the particle energy, zenith and azimuthal angle, time and area. Integrating for energies higher than E, the integral energy spectrum can be obtained:

$$M(>E) = \int_E^\infty D\, dE = \frac{dN}{dA dt d\Omega} (\mathrm{cm}^{-2}\ \mathrm{s}^{-1}\ \mathrm{sr}^{-1}) \tag{3}$$

The muon spectrum has been experimentally measured and shown to vary significantly with energy and zenith angle. The most representative experimental measurements are compiled in Table 1. The experiments cover a wide range of energies, from 0.2 to 20,000 GeV, zenith angles from 0° to 89° and 10 m to 1270 m altitude. It appears that the majority of the experiments were performed in low altitude and vertical or near vertical direction and only few experiments were realized in higher zenith angles. The

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[1] The muon generator is available on the MATLAB file exchange webpage: http://www.mathworks.com/matlabcentral/fileexchange/51203-muon-generator and http://www.mathworks.com/matlabcentral/fileexchange/51257-geant4-muon-energy-and-angular-histogram-generation.