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Development of a boundary magnetic charge method for computing magnetic fields in a system containing saturated magnetic materials

H. Murata*, M. Ishigami, H. Shimoyama

Faculty of Science and Technology, Meijo University, 1-501 Shiogamaguchi, Tempaku-ku, Nagoya 468-8502, Japan

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ABSTRACT

In previous research, we developed a three-dimensional (3D) boundary magnetic charge method (BMCM) for high-accuracy field calculations in a static magnetic field, even when there exist great differences between the magnitudes of permeability between neighboring magnetic materials. This method, however, cannot be applied to a system that contains saturated magnetic materials. In the present study, therefore, we have developed a novel method that addresses this issue. According to this new method, we divide the region containing the magnetic material into small-volume elements and divide the boundaries between neighboring small-volume elements into small-surface elements, assigning each element an appropriate initial value of permeability. The magnetic field inside and outside of the magnetic material is calculated using this permeability. The value of the permeability of each element is iteratively updated using μ–H data. The updated value of the permeability after the i-th iteration, μ_i, is compared with that of the previous value, μ_{i-1}. If the difference between the two values is within a preset range, the iteration process is judged to have converged and the value of μ_i is regarded as the final converged value of the permeability. The magnetic field at an arbitrary point in space and/or inside the body of the magnetic material is calculated from the converged permeability of each element. As a result, we have succeeded in developing a novel BMCM for the calculation of a static magnetic field with high accuracy in a system containing saturated magnetic materials.

1. Introduction

The boundary magnetic charge method (BMCM) is used for calculating a static magnetic field and is based on the fact that there exists a formal correspondence between a static magnetic field and an electrostatic field. This means that the calculation of a magnetic field by BMCM completely parallels the calculation of an electric field by the boundary charge method (BCM) [1–3].

The BCM can be regarded as a boundary element method (BEM). BEMs have distinct advantages over finite element methods (FEMs) in that they give numerically smooth solutions with excellent accuracy and can be used to treat an open-boundary system. Therefore, as an application of electrostatic field calculations, BEMs have been widely used for analyzing the characteristics of electron guns [4,5]. In contrast, the BEM for a static magnetic field has not come into wide use, and is mentioned in only a limited number of references [6–9].

The BMCM is also yet to be widely used for calculating the magnetic field, except for a few examples that treat magnetic deflection yokes [10–12]. The reason might be ascribed to the poor accuracy and long computation time of BMCM, which are mainly caused by numerical double-integration.

We have recently developed a three-dimensional (3D) BMCM for high-accuracy field calculations in a static magnetic field, even when there exists a great difference between the magnitudes of the permeability of neighboring magnetic materials [13]. As a typical example of this calculation, we have performed electron optical analysis on a magnetic-field-superposed objective cathode lens using both BCM and BMCM [14]. The conventional BMCM, however, cannot be applied to a system containing saturated magnetic materials.

Kasper treated a saturated magnetic lens using BEM for the first time [15]. However, the numerical accuracy of the approximate method used in Ref. [15] was found to be inadequate for some saturated lenses. On the other hand, although calculations on saturated magnetic lenses using FEM have been frequently undertaken [16–19], these calculations have all been two-dimensional analyses using the rotationally symmetric magnetic lenses. For a general 3D magnetic field to be calculated using FEM, the entire 3D space and the magnetic materials must be divided into massive finite elements with tetrahedral or hexahedral meshes. In BMCM, on the other hand, the 3D space need not be

* Corresponding author. Tel.: +81 52 832 1151x5109; fax: +81 52 832 1235.
E-mail address: hkmurata@meijo-u.ac.jp (H. Murata).

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divided, i.e., only the surface or boundary of the magnetic material is divided. The number of divisions needed in BMCM is one order of magnitude smaller than that needed in FEM, and this significantly decreases the computational load. Moreover, because the division procedure is very simple, additional software, such as a mesh generator or a mesh builder, are not required in BMCM, although they are required in FEM.

In this study, therefore, we have developed a novel method that is applicable to a static magnetic field containing saturated magnetic materials.

2. The principle of BMCM for computing magnetic fields in saturated magnetic materials

Consider a system in which a magnetic material of high permeability μ_H is placed in a vacuum and magnetized by an external steady current flowing near the magnetic material, as shown in Fig. 1. Such a system is hereafter referred to as a static magnetic field system. The magnetic field at an arbitrary point in space can be expressed as the sum of the magnetic field $\mathbf{H}_C$ caused by the external current and the magnetic field $\mathbf{H}_M$ caused by the magnetic material magnetized by the current, where the relation rot $\mathbf{H}_M=0$ holds true under the condition that there are no currents inside the magnetic material. In this case, we have a formal correspondence between a static magnetic field and an electrostatic field, as shown in Table 1, where we assume that there is no charge in space. These circumstances allow us to utilize BMCM to calculate the magnetic field $\mathbf{H}_M$ caused by the material magnetized by the external current. For the calculation of the magnetic field $\mathbf{H}_C$ caused by the external current, we utilize the Biot–Savart law. The resultant total magnetic field, $\mathbf{H}$, at an arbitrary point in space is expressed as $\mathbf{H}=\mathbf{H}_C+\mathbf{H}_M$.

2.1. Outline of BMCM for computing magnetic fields in saturated magnetic materials

The first step in this method is to divide the entire region of the static magnetic field into two sub-regions: the region L containing a vacuum (i.e., the magnetic medium of low permeability μ_L) and the region H containing the magnetic material of high permeability μ_H. The boundary of division is selected so as to coincide with the surface of the magnetic material of high permeability μ_H, and is hereafter called the surface of the magnetic material, or the interface between μ_L and μ_H, or simply the interface.

In a static magnetic field system that does not contain any saturated magnetic material, it is sufficient to divide only the surface of the magnetic material into small-surface elements, because the value of permeability is treated as a constant value

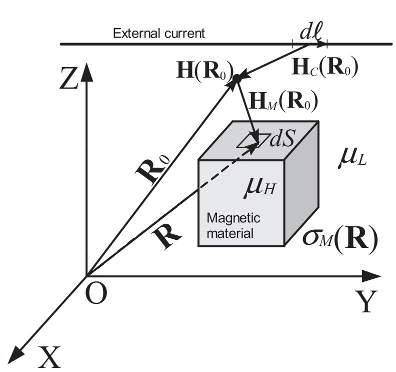

Fig. 1. A system in which a magnetic material of high permeability μ_H is placed in a vacuum of permeability μ_L and magnetized by an external steady current flowing near the magnetic material.

Table 1
Formal correspondence between the static magnetic field and the electrostatic field under the condition that there are no currents inside the magnetic material and that there are no charges in space.

	Electrostatic field	Static magnetic field
Potential	ϕ	ϕ_M
Field	$\mathbf{E}$	$\mathbf{H}_M$
Flux density	$\mathbf{D}$	$\mathbf{B}_M$
Charge density	σ	σ_M
Media	Permittivity ε	Permeability μ
Relations	rot $\mathbf{E}=0$	rot $\mathbf{H}_M=0$
	div $\mathbf{D}=0$	div $\mathbf{B}_M=0$

anywhere inside the magnetic material. In a system in the presence of saturated magnetic materials, however, the body of the magnetic material must further be divided into small-volume elements, because the value of permeability inside the magnetic material is not constant and is different from point to point inside the magnetic material according to the magnitude of the magnetic field at each point. The boundaries between neighboring small-volume elements are further divided into smaller surface elements, and are hereafter called the boundaries inside the magnetic material.

We then arrange magnetic charge densities on the surface of the magnetic material and on the boundaries inside the magnetic material. This is done in such a way that the magnetic charge densities $(\sigma_M)_L$ and $(\sigma_M)_H$ are arranged on the μ_L and μ_H sides of the interface, respectively. The double-layer arrangement of magnetic charge densities is very efficient for high-accuracy numerical calculations compared with the single layer arrangement, especially in the case where there exists a great difference in the magnitude of permeability between neighboring magnetic materials. We assign a magnetic charge density $(\sigma_M)_H$ to the boundaries inside the magnetic material. Note that the magnetic charge densities must be assigned not only to both sides of the surface of the magnetic material but also to the boundaries inside the magnetic material. Further details of the assignment of magnetic charge densities will be described later.

These magnetic charge densities can be considered to appear as a result of the magnetization of the material caused by the external current. The static magnetic field can equivalently be replaced by an electrode system in a vacuum by introducing these magnetic charge densities and using a formal correspondence between the static magnetic field and electrostatic field, as shown in Table 1. The key point of our BMCM is that when we calculate the magnetic field distributions in region L, for example, we use the magnetic charge densities only in region L and do not use them in region H. When we calculate the magnetic field distributions in region H, a similar rule is applied.

The magnetic potential $\phi_L(\mathbf{R}_0)$ at an arbitrary point $\mathbf{R}_0$ in region L (vacuum) is expressed as the sum of the magnetic potential caused by the magnetic charge density $(\sigma_M)_L(\mathbf{R})$ on the vacuum side of the interface and the magnetic potential $\phi_C(\mathbf{R}_0)$ caused by the external current flowing near the magnetic material:

$$\phi_L(\mathbf{R}_0)=\phi_C(\mathbf{R}_0)+\frac{1}{4\pi\mu_0}\iint_{S_L}\frac{(\sigma_M)_L(\mathbf{R})}{|\mathbf{R}-\mathbf{R}_0|}dS. \tag{1}$$

Here, μ_0 $(=\mu_L)$ is the permeability of a vacuum, and S_L is the surface area of the magnetic material. The integration of Eq. (1) is performed over the entire surface area S_L, with the point $\mathbf{R}_0$ being fixed. The magnetic field $\mathbf{H}_L(\mathbf{R}_0)$ at the point $\mathbf{R}_0$ in region L (vacuum) is expressed in terms of the gradient of the magnetic