Composites Part B 83 (2015) 142–152

Contents lists available at ScienceDirect

Composites Part B

journal homepage: www.elsevier.com/locate/compositesb

Refined and generalized hybrid type quasi-3D shear deformation theory for the bending analysis of functionally graded shells

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ARTICLE INFO

Article history:
Received 16 February 2015
Received in revised form
24 April 2015
Accepted 6 August 2015
Available online 24 August 2015

Keywords:
A. Plates
B. Elasticity
C. Analytical modeling

ABSTRACT

The closed-form solution of a generalized hybrid type quasi-3D higher order shear deformation theory (HSDT) for the bending analysis of functionally graded shells is presented. From the generalized quasi-3D HSDT (which involves the shear strain functions "$f(\zeta)$" and "$g(\zeta)$" and therefore their parameters to be selected "m" and "n", respectively), infinite six unknowns' hybrid shear deformation theories with thickness stretching effect included, can be derived and solved in a closed-from. The generalized governing equations are also "m" and "n" parameter dependent. Navier-type closed-form solution is obtained for functionally graded shells subjected to transverse load for simply supported boundary conditions. Numerical results of new optimized hybrid type quasi-3D HSDTs are compared with the first order shear deformation theory (FSDT), and other quasi-3D HSDTs. The key conclusions that emerge from the present numerical results suggest that: (a) all non-polynomial HSDTs should be optimized in order to improve the accuracy of those theories; (b) the optimization procedure in all the cases is, in general, beneficial in terms of accuracy of the non-polynomial hybrid type quasi-3D HSDT; (c) it is possible to gain accuracy by keeping the unknowns constant; (d) there is not unique quasi-3D HSDT which performs well in any particular example problems, i.e. there exists a problem dependency matter.

1. Introduction

Laminated composite structures such as shells are extensively used in the industry. However, even the designer's effort to tailor different laminate properties to suit a particular application; some laminated composite structures suffer from discontinuity of material properties at the interface of the layers and constituents of the composite. Therefore the stress fields in these regions create interface problems and thermal stress concentrations under high temperature environments. Furthermore, large plastic deformation of the interface may trigger the initiation and propagation of cracks in the material [1]. In order to alleviate this problem, functionally graded materials (FGMs) were proposed by Bever and Duwez [2]. Then, this kind of materials were developed and successfully used in industrial applications since 1984 [3].

In the most general case, FGMs are materials with spatial variation of the material properties. However, in most of the applications reported in the literature, as in the present work, the variation is only along the thickness; demonstrating the present state of development of FGMs.

Recently, several researchers have reported results on functionally graded plates (FGPs) and shells. Both analytical and numerical solutions for these cases can be found in the literature, Birman and Byrd [4], see also Mantari and Guedes Soares [5–9]. An updated literature review of FGMs can be found in the work by Jha et al. [10]. In the present article, the relevant and recent related work on functionally graded shells is described in what follows.

Readers interested in developing HSDTs should consult the referential papers by Reddy [11], Vel and Batra [12,13] and Cheng and Batra [14]. Alternative bibliography can be also the paper by Liew et al. [15] on postbuckling analysis of functionally graded cylindrical shells under axial compression and thermal loads using the element-free kp-Ritz method. In this paper, the authors developed the formulation to handle problems of small strains and moderate rotations, based on the FSDT for shells and von Kármán strains.

Sofiyev and Kuruoglu [16] studied the torsional vibration and buckling of cylindrical shells with FG coatings surrounded by an elastic medium. Consequently, Sofiyev and Kuruoglu [17] presented a theoretical approach to solve vibration problems of FG truncated

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http://dx.doi.org/10.1016/j.compositesb.2015.08.048
1359-8368/

conical shells under mixed boundary conditions. Then, Sofiyev [18] investigated the dynamic instability of exponentially graded sandwich cylindrical shells under static and time dependent periodic axial loadings using HSDT.

Deniz [19] studied the response of an FG coated truncated conical shell subjected to an axial load. The author performed the analysis through non-linear equations governing the finite deformations of the shell. Ghannad et al. [20] presented an analytical solution for deformations and stresses of axisymmetric clamped–clamped thick cylindrical shells with variable thickness made of FGMs subjected to internal pressure. The authors used the FSDT and matched asymptotic method (MAM) of the perturbation theory.

Fraldi and colleagues [21] presented the exact analytical solutions for the elastic response of a solid circular cylinder composed by the assembly of a central core and n surrounding hollow phases, all made of different homogeneous elastic materials, under de Saint Venant load conditions. The authors obtained an equivalent one-dimensional homogenized beam model for the whole object. Tornabene et al. [22] performed an extensive study on doubly-curved FG shells structures using CUF and the Murakami's Zig-Zag (ZZ) function. The authors used the Generalized Differential Quadrature (GDQ) method and good referential results were obtained.

Xie et al. [23] used the Haar Wavelet Discretization (HWD) method-based solution approach to study the free vibration analysis of FG spherical and parabolic shells of revolution with arbitrary boundary conditions along with the FSDT. Kim [24] studied the free vibration characteristics of FG cylindrical shells partially resting on elastic foundation with an oblique edge by using the FSDT. Shooshtari and Razavi [25] studied, analytically and by using the FSDT, the linear and nonlinear free vibration of symmetrically laminated magneto-electro-elastic doubly-curved thin shell resting on an elastic foundation. Qu et al. [26] described a general formulation for free, steady-state and transient vibration analyses of FG shells of revolution subjected to arbitrary boundary conditions. The formulation is derived by means of a modified variational principle in conjunction with a multi-segment partitioning procedure on the basis of the FSDT. Thai and Kim [27] performed a remarkable review on equivalent single layer theories (ESL) in the modeling of functionally graded plates and shells.

Carrera and co-workers [28] studied the static analysis of FGPs and shells. The stretching effect was included in the mathematical formulation and the importance of the transverse normal strain effects in the mechanical prediction of stresses of FGPs and shells was remarked. Neves et al. [29,30] and Ferreira et al. [31] presented a quasi-3D hybrid type (polynomial and trigonometric) shear deformation theory for the static and free vibration analysis of functionally graded plates by using meshless numerical method. Their formulation can be seen as a generalization of the original Carreras's Unified Formulation (CUF), by introducing different non-polynomial displacement fields for in-plane displacements, and polynomial displacement field for the out-of-plane displacement. Mantari and Guedes Soares [5–9] presented bending results of FGM by using new non-polynomial HSDTs. In Refs. [7] and [8], the stretching effect was included and improved results of displacement and in plane normal stresses compared with [5] and [6] were found. Recently, the authors developed an accurate and attractive optimized quasi-3D HSDTs for advanced composite plates and shells [32,33] by utilizing the well-known polynomial sinusoidal shear strain shape function.

In the other hand, on the basis of the 4-unknown plate theory and polynomial shear strain shape function, Abdelaziz et al. [34] studied the static analysis of FG sandwich plates. Consequently, Mechab et al. [35] considered the static and dynamic analysis of FGPs with new non-polynomial shear strain shape function (hyperbolic). Recently, a 5 unknown variables trigonometric plate theory (TPT) with stretching effect was developed by Thai and Kim [36] showing good accuracy with respect to its counterpart the TPT with 6-unknowns.

Looking for generalized formulations of shear deformation theories for classical and advanced composites, it can be said that they are rare in the literature. Regarding to generalized formulations in classical composites, it is important to remark the work done by Soldatos [37], Carrera (CUF) [38] and Demasi [39,40], Mantari and Guedes Soares [41].

Besides the powerful CUF there exists a generalized formulation proposed by Zenkour [42], which were extended to cover the stretching effect in Zenkour [43]. Matsunaga [44] also developed a generalized HSDT based on polynomial shear strain shape functions. In Mantari and Guedes Soares [8], a generalized hybrid type HSDT for plates was developed. This generalized theory is able to reproduce the theory proposed in Refs. [7,43] and others as special case.

The generalized HSDT of functionally graded plates presented by Zenkour [42,43] is similar to the one formulated by Soldatos [37] for laminated composites. Normally non-polynomial share strain shape functions, such as trigonometric, trigonometric hyperbolic, exponential, etc., can be used in this type of generalized formulation, see also Mantari and Guedes Soares [41]. However, the thickness expansion model ($g(z)$) is conditioned by the in-plane displacement model ($f(z)$), i.e. the transverse shear strain function is an even function which is the derivative of the in-plane shear strain shape function ($g(z)=f'(z)$). Therefore, there is no freedom in choosing the shear strain shape functions, i.e. the through the thickness displacement field modeling.

The present formulation has that freedom, i.e. $g(z)$ can be $f'(z)$ or different transverse shear strain function, and therefore infinite hybrid type shear deformation theories (polynomial or non-polynomial or hybrid type) can be created just having 6 unknown variables or 6 degree of freedom (DOF) for finite element analysis.

The generalized hybrid type HSDT for plates and shells presented here, and for example the well-known CUF (extended to include non-polynomial shape strain functions in their formulation [29–31]) demand the development of new non-polynomial shape strain functions, which can be adapted to this advanced generalized formulation perhaps for better performance. In the present work, in addition to the main contribution of the present paper, new non-polynomial shape strain functions are presented for the first time.

It is important to remark that the static or bending problem of *shells* made of FGMs are not much explored neither available in the literature with the exception of CPT formulations (in the case of *plate* bending problems based on shear deformation theories, the contribution is quite representative), it may be because not much attention were given to the static behavior of FGMs compared with thermomechanical behavior which initially was the main concern due to the application requirements. Moreover, the non-polynomial function based-HSDTs are not widely used compared with the polynomial function based-HSDTs except for the case of the sinusoidal shear deformation theory (SSDT) as reported by Thai and Kim [27] who performed an interesting review on shells and functionally graded materials in the context of equivalent single layer and PVD and RMVT variational statements. In both directions, this paper contributes with the implementation of new quasi-3D non-polynomial hybrid type HSDTs to study the bending problem of single and sandwich shells. The SSDT with 5 and 6 unknown variables were previously optimized by this author and Guedes Soares [32,33]. In Ref. [32] the bending shell problem were studied through elegant optimized SSDT.

In this paper a generalized quasi-3D hybrid type HSDT for shells having fixed number of unknowns, 6 in this case, is formulated and