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Static and free vibration analysis of functionally graded plates based on a new quasi-3D and 2D shear deformation theories

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ABSTRACT

In this study, two dimensional (2D) and quasi three-dimensional (quasi-3D) shear deformation theories are presented for static and free vibration analysis of single-layer functionally graded (FG) plates using a new hyperbolic shape function. The material of the plate is inhomogeneous and the material properties assumed to vary continuously in the thickness direction by three different distributions; power-law, exponential and Mori–Tanaka model, in terms of the volume fractions of the constituents. The fundamental governing equations which take into account the effects of both transverse shear and normal stresses are derived through the Hamilton's principle. The closed form solutions are obtained by using Navier technique and then fundamental frequencies are found by solving the results of eigenvalue problems. In-plane stress components have been obtained by the constitutive equations of composite plates. The transverse stress components have been obtained by integrating the three-dimensional stress equilibrium equations in the thickness direction of the plate. The accuracy of the present method is demonstrated by comparisons with the different 2D, 3D and quasi-3D solutions available in the literature.

1. Introduction

Functionally graded materials (FGMs) are a type of advanced composite materials whose properties vary gradually and continuously from one surface to another. The mechanical properties of FGM vary along the thickness direction in the material depending on a function. Due to this feature, the FGMs have some advantages such as eliminating the material discontinuity and avoiding the delamination failure, reducing the stress levels and deflections. Combination of these characteristics attracts application of FGMs in many engineering fields from biomedical to civil engineering.

In recent years, FGM applications have received considerable increase. The increase in FGM applications requires accurate models to predict their response. The static and dynamic behavior of FGM plates have been analyzed and reported by many researchers in recent ten years. Kashtalyan [1] developed a three-dimensional elasticity solution for a simply supported FG plate subjected to transverse loading. Tornabene et al. [2] analyzed the dynamic behavior of FG conical, cylindrical shells and annular plates. They used the first-order shear deformation theory and the linear elasticity theory to analyze moderately thick structural elements. Matsunaga [3] presented a two-dimensional higher-order theory for the evaluation of displacement and stresses in FGM plates subjected to thermal and mechanical loadings. The theory can take into account the effects of both transverse shear and normal stresses. Zhao et al. [4] presented a free vibration analysis of metal and ceramic FG plates that uses the element-free kp-Ritz method. The first-order shear deformation plate theory is employed to account for the transverse shear strain and rotary inertia, and mesh-free kernel particle functions are used to approximate the two-dimensional displacement fields. Vaghefi et al. [5] developed a version of meshless local Petrov–Galerkin method to obtain 3D static solutions for thick FG plates. Orakdogen et al. [6] studied the coupling effect of extension and bending in FG plate subjected to transverse loading for Kirchhoff-Love plate theory equations. Tamijani and Kapania [7] developed a element free Galerkin method for the free vibration of a FG plate with curvilinear stiffeners. The governing equations for the plate and stiffeners are derived by using the first order shear deformation theory. Gunes

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et al. [8] carried out an experimental work about the low-velocity impact behavior of FG clamped circular plates. Zhang et al. [9] presented a 3D elasticity solution for static bending of thick FG plates using a hybrid semi-analytical approach-the state-space based differential quadrature method. Filippi et al. [10] used the 1D Carrera Unified Formulation to perform static analyses of FG structures. Swaminathan et al. [11] presented a comprehensive review of the various methods employed to study the static, dynamic and stability behavior of FGM plates. Barretta et al. [12] analyzed the torsion of linearly elastic isotropic beams, with both cross-sectional and axial inhomogeneities. New closed-form solutions are found in the present paper, by detecting axial distributions of longitudinal and shear moduli inducing an axially uniform warping field. Sofiyev and Kuruoglu [13] presented a theoretical approach to solve vibration problems of FG truncated conical shells under mixed boundary conditions. Zafarmand and Kadkhodayan [14] investigated the three dimensional static and dynamic behavior of a thick sector plate made of two-directional FGMs. Zhu and Liew [15] presented free vibration analyses of metal and ceramic FG plates with the local Kriging meshless method.

It is obvious that non-negligible shear deformations occur at the thick and moderately thick plates and the classical plate theory shows inaccurate results. So, transverse shear deformations have to be taken into account in the analysis. There are numerous plate theories that include transverse shear strains for the advanced composites like FGMs. Zenkour [16] analyzed the static response of simply supported FG rectangular plates subjected to transverse uniform load. Bodaghi and Saidi [17] presented an analytical approach which converts the coupled governing stability equations into two uncoupled partial differential equations in terms of transverse displacement for buckling analysis of thick FG rectangular plates. Benachour et al. [18] presented a four variable refined plate theory for free vibration analysis of plates made of FGMs with an arbitrary gradient. Hosseini-Hashemi et al. [19] presented a new exact closed-form procedure to solve free vibration analysis of FG rectangular thick plates based on the Reddy's third-order shear deformation plate theory while the plate has two opposite edges simply supported. Thai and Kim [20] developed a new shear deformation theory for bending and free vibration analysis of FG plates. Tran et al. [21] presented a novel and effective formulation based on isogeometric approach and higher-order deformation plate theory to study the behavior of FGM plates. Mechab et al. [22] presented the analytical solutions of static and dynamic analysis of FG plates using a four-variable refined plate theory. Mantari and Soares [23] presented static response of FG plates by using a recently developed higher order shear deformation theory.

The thickness stretching effect is ignored in the most of shear deformation theories by assuming the transverse displacement as constant. This assumption is inaccurate especially for thick FGM plates. To overcome this problem, some quasi-3D theories presented in the literature. Zenkour [24] considered the bending problem of transverse load acting on an isotropic inhomogeneous rectangular plate using both 2D trigonometric and 3D elasticity solutions. Matsunaga [25] analyzed the natural frequencies and buckling stresses of plates made of FGMs by taking into account the effects of transverse shear and normal deformations and rotatory inertia. Carrera et al. [26] evaluated the effect of thickness stretching in plate/shell structures made by materials which are FGM in the thickness directions. Neves et al. [27] presented a new application for Carrera's unified Formulation (CUF) to analyze FG plates. Neves et al. [28] presented an original hyperbolic sine shear deformation theory which account for through the thickness deformations for the bending and free vibration analysis of FG plates. Zenkour [29] derived a refined trigonometric higher-order plate theory which the effects of transverse shear strains as well as the transverse normal strain are taken into account for bending analysis of FGM plates. Jha et al. [30] presented a free vibration response of FG elastic, rectangular, and simply (diaphragm) supported plates based on higher order shear/shear-normal deformations theories. Sheikholeslami and Saidi [31] studied the free vibration of simply supported FG rectangular plates resting on two-parameter elastic foundation using the higher-order shear and normal deformable plate theory of Batra and Vidoli by an analytical approach. Thai and Kim [32] developed a quasi-3D sinusoidal shear deformation theory for FG plates. Mantari and Soares [33] presented the analytical solutions to the bending analysis of FGM plates by using a new trigonometric higher order and hybrid quasi-3D shear deformation theories. Hebali et al. [34] presented a new quasi-3D hyperbolic shear deformation theory for bending and free vibration analysis of FG plates. Thai and Choi [35] improved a refined pate theory to account for the effect of thickness stretching in FG plates. Mantari and Soares [36] presented a quasi-3D trigonometric shear deformation theory for the bending analysis of FG plates. Belabed et al. [37] presented an efficient and simple higher order shear and normal deformation theory for FGM plates. Alijani and Amabili [38] investigated the nonlinear forced vibrations of moderately thick FG rectangular plates by considering higher-order shear deformation theories that take into account the thickness deformation effect.

In this study, a new quasi-3D hyperbolic shear deformation theory is developed for static and free vibration analysis of FG plates. The proposed theory gives good accuracy and accounts for a parabolic transverse shear deformation shape function and satisfies shear stress free boundary conditions of top and bottom surfaces of the plate without using shear correction factors. Besides, the theory accounts for the thickness stretching effect by using the same hyperbolic function. Governing equations are derived from the Hamilton's principle. Navier solution is used to obtain the closed form solutions for simply supported functionally graded plates. The in-plane stresses are calculated from the linear constitutive relations and the transverse shear and normal stresses are obtained by integrating the three dimensional stress equilibrium equations of elastic media by satisfying the stress boundary conditions on the top and bottom surfaces of a plate. Numerical examples are presented to illustrate the accuracy and efficiency of the present theory.

2. Fundamental formulations

2.1. Homogenization models for continuous gradation

Consider a single-layer rectangular plate, having uniform thickness h, length a, width b and made of a functionally graded material (Fig. 1). In this study, the compositions and volume fractions of the constituents in FGM are considered to vary gradually through the thickness according to: a. The power-law distribution, b. The exponential distribution, c. The Mori–Tanaka homogenization model. Since the effects of Poisson's ratio ν on the response of FG plates are very small, it is assumed to be constant for all gradation models.

a. The power-law (P-FGM) distribution

The volume fraction of the P-FGM plate is assumed to vary continuously through the thickness of the plate in according to the power law distribution (Bao and Wang [39]) as follows:

$$P(z) = P_m + (P_m - P_c)\left(\frac{1}{2}+\frac{z}{h}\right)^k \tag{1}$$