Composites Part B 83 (2015) 226–241

Contents lists available at ScienceDirect

Composites Part B

journal homepage: www.elsevier.com/locate/compositesb

Nonlinear analysis of forced vibration of nonlocal third-order shear deformable beam model of magneto–electro–thermo elastic nanobeams

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A R T I C L E I N F O

Article history:
Received 29 December 2014
Received in revised form
9 May 2015
Accepted 7 August 2015
Available online 20 August 2015

Keywords:
A. Nano-structures
B. Vibration
C. Numerical analysis

A B S T R A C T

This paper deals with the forced vibration behavior of nonlocal third-order shear deformable beam model of magneto–electro–thermo elastic (METE) nanobeams based on the nonlocal elasticity theory in conjunction with the von Kármán geometric nonlinearity. The METE nanobeam is assumed to be subjected to the external electric potential, magnetic potential and constant temperature rise. Based on the Hamilton principle, the nonlinear governing equations and corresponding boundary conditions are established and discretized using the generalized differential quadrature (GDQ) method. Thereafter, using a Galerkin-based numerical technique, the set of nonlinear governing equations is reduced into a time-varying set of ordinary differential equations of Duffing type. The pseudo-arc length continuum scheme is then adopted to solve the vectorized form of nonlinear parameterized equations. Finally, a comprehensive study is conducted to get an insight into the effects of different parameters such as nonlocal parameter, slenderness ratio, initial electric potential, initial external magnetic potential, temperature rise and type of boundary conditions on the natural frequency and forced vibration characteristics of METE nanobeams.

1. Introduction

A new class of smart materials called magneto–electro–elastic (MEE) has recently received significant attention from research communities as a result of their novel magneto–electric coupling effects [1]. These composite materials are comprised of the piezoelectric phase and piezomagnetic phase and are capable of converting energy amongst three forms, namely electric, magnetic and elastic [2]. Compared to MEE bulk composite materials, MEE nanomaterials possess higher magnetoelectric coupling [3]. Having novel magnetoelectric coupling effects together with their smaller size and larger surface to volume ratio have made MEE materials very promising for broad potential applications in many technological fields such as vibration control, sensor and actuator applications, structural health monitoring, robotics, energy harvesting, medical instruments and so on [4–6].

During the two past decades, considerable effort has been put into estimating the mechanical characteristics of nanostructures [7–12]. Many experimental and atomistic simulations reported that the properties of MEE materials are size-dependent [13–15]. Thus, it is of crucial importance to take the size effect into consideration in theoretical and experimental studies concerning with MEE nanostructures. Since the classical continuum theory fails to capture the size effects of nanostructures, different types of higher-order continuum theories have been developed [16–20]. The nonlocal elasticity theory proposed by Eringen [21,22] is one of these non-classical theories which has been widely applied to analyze the size effects of different nanostructures such as nanowires and nanorods [23–25], single- and multi-walled carbon nanotubes [26–29], graphene sheets and nanoplates [30–33], mass sensors [34], nano-peapods [35], nanobeams [36,37] and so forth. The nonlocal elasticity theory has been also employed to explore the size-dependent mechanical behavior of piezoelectric nanostructures [38–40]. In this regard, Ke and Wang [41] and Ke et al. [42] made the first attempt to study the thermo-electro-

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http://dx.doi.org/10.1016/j.compositesb.2015.08.038

mechanical linear and nonlinear vibration of piezoelectric nanobeams through the use of nonlocal Timoshenko beam theory. Utilizing nonlocal theory in conjunction with Kirchhoff theory, Liu et al. [43] analyzed the thermo-electro-mechanical free vibration of piezoelectric nanoplates. They demonstrated that the natural frequencies of these structures are very sensitive to the electro-mechanical loadings and insensitive to the thermal loading. Asemi et al. [44] by developing a nonlinear continuum model investigated the large amplitude vibration of nano-electromechanical resonators using piezoelectric nanofilms under external electric voltage. On the basis of nonlocal Timoshenko beam theory along with von Kármán geometric nonlinearity, Liu et al. [45] studied the buckling and postbuckling of size-dependent piezoelectric nanobeams under thermo-electro-mechanical loadings. Ke et al. [46] studied the free vibration of piezoelectric nanoplates under various edge supports by proposing a nonlocal Mindlin plate model. Moreover, the thermo-electro-mechanical vibration of piezoelectric cylindrical nanoshells with different boundary conditions was investigated by Ke et al. [47] using a nonlocal Love thin shell model. Besides the aforementioned studies, a few investigations have been also carried out in the open literature dealing with the surface effect on the piezoelectric nanostructures [48–51].

More recently, using the nonlocal elasticity theory, a few studies have been performed on the size-dependent vibration and buckling behaviors of MEE nanostructures. Ke and Wang [52] employed the nonlocal theory to explore the free vibration of MEE Timoshenko nanobeams subjected to an external electric potential, a magnetic potential and a constant temperature rise. They found that the natural frequency of MEE nanobeams is not sensitive to temperature rise, while it is highly affected by electric and magnetic loadings. Li et al. [53] using the Mindlin plate theory and ignoring the in-plane electric and magnetic fields analyzed the buckling and free vibration of MEE nanoplates resting on a Pasternak foundation. Ke et al. [54] studied the free vibration of MEE nanoplates on the basis of nonlocal Kirchhoff plate theory. In another study, the same authors [55] developed an embedded MEE cylindrical nanoshell model based on the nonlocal Love's shell theory. Their study revealed that the fundamental frequency of MEE nanoshells is quite sensitive to thermo-electro-magnetic loadings and also decreases with increasing the length-to-radius ratio. Razavi and Shooshtari [56] based on the first-order shear deformation theory together with von Kármán nonlinear strains studied the nonlinear free vibration of MEE rectangular plate under simply supported boundary condition. They demonstrated that the nonlinear frequency ratio decreases through using MEE layers in composite plates.

In this study, we made the first attempt to analyze the forced vibration behavior of nonlocal third-order shear deformable beam model of magneto–electro–thermo elastic (METE) nanobeams using the Eringen's nonlocal theory and the von Kármán geometric nonlinearity. It is assumed that the METE nanobeam is subjected to the external electric potential, magnetic potential and uniform temperature rise. Hamilton principle is first employed to formulate the nonlinear governing equations and corresponding boundary conditions which are then discretized through the generalized differential quadrature (GDQ) method. Afterwards, a Galerkin-based numerical technique is adopted to reduce the set of nonlinear governing equations into a time-varying set of ordinary differential equations of Duffing type. The pseudo-arc length continuum method is then employed to solve the vectorized form of nonlinear parameterized equations. Finally, a detailed parametric study is carried out to highlight the effects of nonlocal parameter, slenderness ratio, initial electric potential, initial external magnetic potential and temperature rise on the natural frequency as well as forced vibration behavior of METE nanobeams under different end supports.

2. Nonlocal theory for magneto–electro–elastic materials

Eringen's nonlocal elasticity theory [21,22] assumes that the stress at a reference point is a function of the strain field at every point in the body. This observation accords with both the experiment of the phonon dispersion and the atomic theory of the lattice dynamics [22]. According to the nonlocal elasticity theory, some phenomena associated with atomic and molecular scales including high frequency vibration and wave dispersion can be satisfactorily described. This theory provides information about the forces between atoms and the internal length scale as a material parameter is introduced into the constitutive equations. From a mathematical aspect, the basic equations for a homogenous and nonlocal magneto–electro–thermo-elastic solid with zero body force can be expressed as

$$\sigma_{ij} = \int_V \lambda\left(\left|x - x'\right|, \mu\right)\left[c_{ijkl}\varepsilon_{kl}\left(x'\right) - e_{mij}E_m\left(x'\right) - q_{nij}H_n\left(x'\right) - \beta_{ij}\Delta T\right] dV\left(x'\right), \quad \forall x \in V \tag{1a}$$

$$D_i = \int_V \lambda\left(\left|x - x'\right|, \mu\right)\left[e_{ikl}\varepsilon_{kl}\left(x'\right) + s_{im}E_m\left(x'\right) + d_{in}H_n\left(x'\right) + p_i\Delta T\right] dV\left(x'\right), \quad \forall x \in V \tag{1b}$$

$$B_i = \int_V \lambda\left(\left|x - x'\right|, \mu\right)\left[q_{ikl}\varepsilon_{kl}\left(x'\right) + d_{im}E_m\left(x'\right) + \mu_{in}H_n\left(x'\right) + \lambda_i\Delta T\right] dV\left(x'\right), \quad \forall x \in V \tag{1c}$$

where σ_{ij}, ε_{ij}, D_i, E_i, B_i, and H_i respectively symbolize the stress, strain, electric displacement, electric field, magnetic induction and magnetic field; c_{ijkl}, e_{mij}, s_{im}, q_{nij}, d_{ij}, s_{ij}, p_i and λ_i denote the elastic, piezoelectric, dielectric constants, piezomagnetic, magnetoelectric, magnetic, pyroelectric and pyromagnetic constants, respectively; β_{ij}, ΔT and ρ signal the thermal moduli, temperature change and mass density, respectively. Furthermore, $\lambda(|x-x'|,\mu)$ denotes the nonlocal attenuation function that incorporates into the constitutive equations at the reference point x generated by the local strain at the source x'; $|x-x'|$ is the Euclidean distance and $\mu = e_0a/l$ is the scale coefficient which incorporates the small scale factor in which e_0 presents a material constant determined experimentally or approximated through matching the dispersion curves of the plane waves with ones of the atomic lattice dynamics; a and l respectively show the internal (e.g. lattice parameter, granular size) and external characteristic lengths (e.g. crack length, wavelength) of the nanostructures.

According to the Eringen's theory, the constitutive equation presented in the spatial integral forms can be converted to the equivalent differential constitutive equations as follows

$$\sigma_{ij} - (e_0a)^2\nabla^2\sigma_{ij} = c_{ijkl}\varepsilon_{kl} - e_{mij}E_m - q_{nij}H_n - \beta_{ij}\Delta T \tag{2a}$$

$$D_i - (e_0a)^2\nabla^2 D_i = e_{ikl}\varepsilon_{kl} + s_{im}E_m + d_{in}H_n + p_i\Delta T \tag{2b}$$