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Effects of angular misalignment on optical klystron undulator radiation

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ABSTRACT

In this paper ,we analyze the important effects of optical klystron undulator radiation with an angular offset of the relativistic electron beam in the second undulator section. An anlytical expression for the undulator radiation is obtained through a transparent and simple procedure.It is shown that the effects of the angular offset is more severe for longer undulator lengths and with higher dispersive field strengths.Both these effects are less pronounced for undulators with large K values.

1. Introduction

In recent years there exists interest in optical klystron undulator for synchrotron radiation sources and free electron lasers [1–6]. The optical klystron undulator consists of two undulator sections known as modulator and radiator separated by a drift or dispersive section. The electrons undergo bunching leading to energy and density modulation in the dispersive section. As a consequence there is enhanced radiation in the radiator due to pre bunching of the electrons and due to the interference of radiation from the two undulator sections as well. There are several successful experiments on FEL with optical klystron undulator concept [7–12]. The issues of free electron laser with optical klystron undulator have been addressed and reported both in symmetrical and asymmetrical configurations. The symmetrical optical klystron undulator consists of two undulators of identical length and K values leading to enhanced gain due to strong interference of the radiations from the undulators [13–17]. The asymmetrical configuration of the optical klystron undulator have been studied with gain reduction [18–21]. The betatron effect in optical klystron and effect of beam energy spread on cascade optical klystron undulator radiation too has been discussed [22–26]. In the optical klystron , a long drift space or a shorter dispersive magnet is placed in between the two undulaor sections . In the long drift space case, the electron –optical interaction is switched off by removing the undulator field. The net changes of the electron phase i.e $\Delta\xi$ is then given by $\Delta\xi = D\upsilon$, where D is the dimensionless time of the drift and υ is the detuning resonance parameter. With a dispersive magnet in between the two undulator sections the magnitude of the field is tuned to turn off the resonance and the change of electron phase is governed by the same relation. A requirement of the dispersive magnet field is that both the first and second field integral in the length of the dispersive section must be zero to ensure zero net transverse and zero angular displacement of the electron at the input of the second undulator section of the optical klystron. Similar condition holds good for the long drift space case where the electron is made to enter the second undulator without any change in its angular or transverse displecement .

In this paper, we remove this restriction and assume that the electron enters the second undulator under an angle (see Fig.(1a) and (b)) . In Section 2 ,we solve the Lienard Wiechert potential analytically in the far field limit. Results are discussed in Section 3. It is shown that beyond a certain angle of injection of the electron in the second undulator , the klystron type spectrum is lost.The effects are more pronounced for longer undulator lengths and higher dispersive field strengths. Both these effects are less pronounced for undulator with large K values.

2. Optical-Klystron Undulator radiation

The brightness i.e. energy radiated per unit solid angle per unit frequency interval is given by the Lienard–Wiechert potential [27]

$$\frac{d^2I}{d\Omega d\omega} = \frac{e^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty}\left\{\hat{n}\times\left(\hat{n}\times\vec{\beta}\right)\right\}\exp\left[i\omega\left(t-\frac{\hat{n}\cdot\vec{r}}{c}\right)\right]dt\right|^2 \tag{1}$$

where e is the electron charge, c is the velocity of the light, ω is the radiation frequency, $\vec{r}$ is the electron trajectory, $\hat{n}$ is an observation unit vector and $\vec{\beta}$ is the normalized electron velocity. In a simple undulator structure, the integral is done over a time from 0 to T where the electron experiences an effective acceleration i.e $T = \frac{L}{c\langle\beta_z\rangle}$, L is the

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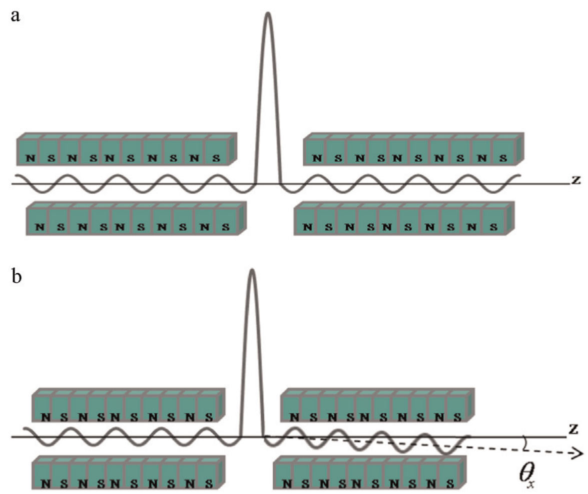

Fig. 1. a. Optical klystron with perfect beam; b.optical klystron with imperfect beam in the Radiator.

undulator length. In a simple undulator structure of length L, we consider $\langle\beta_z\rangle \approx 1$ under relativistic limit and $T = \frac{L}{c\langle\beta_z\rangle} \approx \frac{L}{c}$. We assume that the electron enters the undulator magnetic field whose on-axis field is specified by

$$B = \hat{y}B_u \sin(k_u z) \tag{2}$$

B_u is the peak undulator magnetic field strength, k_u is the undulator field wave number i.e $k_u = \frac{2\pi}{\lambda_u}$, λ_u is the undulator period. Eq. (1) is solved for the undulator field in the vertical $\hat{y}$ direction as in Eq. (2) with $\hat{z}$ is the longitudinal co-ordinate. The electron oscillations are in the $\hat{x}\hat{z}$ plane. For an undulator field arranged with N periods, the interaction time is given by

$$T = \frac{N\lambda_u}{c} = \frac{2N\pi}{\Omega_u},$$

where

$$\Omega_u = k_u c \tag{3}$$

The optical klystron undulator is defined by two undulator sections separated by drift space or an dispersive section. In this case of two identical undulators , the time integral in Eq. (1) is read as

$$\int_0^T dt\,(.....) = \int_0^{\frac{2N\pi}{\Omega_u}} dt\,(.....) + \int_{\frac{2N\pi}{\Omega_u}(1+D)}^{\frac{2N\pi}{\Omega_u}(2+D)} dt\,(.....) \tag{4}$$

In Eq. (4), D is the dimensionless length of the drift section normalized to the undulator length .Using Eq. (4), Eq. (1) can be rewritten as

$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2 c} \times \left| \int_0^{\frac{2N\pi}{\Omega_u}} \left[\hat{n}\times\left(\hat{n}\times\vec{\beta}\right)\right]_{xI} \exp\left\{i\omega\left(t-\frac{z}{c}\right)\right\}_{I} dt + \int_{\frac{2N\pi}{\Omega_u}(1+D)}^{\frac{2N\pi}{\Omega_u}(2+D)} \left[\hat{n}\times\left(\hat{n}\times\vec{\beta}\right)\right]_{xII} \exp\left\{i\omega\left(t-\frac{z}{c}\right)\right\}_{II} dt \right|^2 \tag{5}$$

Introducing the definitions as

$$T_{xI} = \int_0^{\frac{2N\pi}{\Omega_u}} \left[\hat{n}\times\left(\hat{n}\times\vec{\beta}\right)\right]_{xI} \exp\left\{i\omega\left(t-\frac{z}{c}\right)\right\}_{I} dt$$
$$T_{xII} = \int_{\frac{2N\pi}{\Omega_u}(1+D)}^{\frac{2N\pi}{\Omega_u}(2+D)} \left[\hat{n}\times\left(\hat{n}\times\vec{\beta}\right)\right]_{xII} \exp\left\{i\omega\left(t-\frac{z}{c}\right)\right\}_{II} dt \tag{6}$$

Eq. (5) is simplified to read as

$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\left|T_{xI} + T_{xII}\right|^2 \tag{7}$$

For an undulator magnetic field specified by Eq. (2), the electron trajectory is given by Lorentz force equation and is read as

$$\beta_x(t) = -\frac{K}{\gamma}\cos(\Omega_u t)$$
$$\beta_z = \left(1 - \frac{1}{2\gamma^2} - \frac{K^2}{4\gamma^2}\right) - \frac{K^2}{4\gamma^2}\cos(2\Omega_u t)$$
$$z(t) = \left(1 - \frac{1}{2\gamma^2} - \frac{K^2}{4\gamma^2}\right)ct - \frac{K^2 c}{8\gamma^2\Omega_u}\sin(2\Omega_u t) \tag{8}$$

β_x , β_y are the transverse and longitudinal velocity of the electron , $z(t)$ is the longitudinal electron trajectory. It is assumed that the electron enters on axis the magnetic field of the first undulator section, there by evaluation of the finite term in Eq. (6) in the first section of the undulator read as

$$\left(\hat{n}\times\hat{n}\times\vec{\beta}\right)_{xI} = \frac{K}{\gamma}\cos(\Omega_u t)$$
$$\exp\left[i\omega\left(t-\frac{z}{c}\right)\right]_I = \sum_{m=-\infty}^{\infty} \exp\left[\frac{i\omega t}{2\gamma^2}\left(1+\frac{K^2}{2}\right) - im\Omega_u t\right] J_m(0,\xi_1) \tag{9}$$

$J_m(0,\xi_1)$ is the Generalized Bessel function (GBF) with $\xi_1 = -\frac{\omega K^2}{8\gamma^2\Omega_u}$. In this paper it is assumed that the second undulator section is not tilted and the electron enters the second undulator section under an angle θ_x .The radiation is observed under this angle (Fig. (1b)). Then electron velocity trajectory in the second undulator section under this consideration can be written as

$$\beta_x = -\frac{K}{\gamma}\cos(\Omega_u t) + \theta_x$$
$$\beta_z = \left(1 - \frac{1}{2\gamma^2} - \frac{K^2}{4\gamma^2} - \frac{\theta_x^{\,2}}{2}\right) - \frac{K^2}{4\gamma^2}\cos(2\Omega_u t) + \frac{K\theta_x}{\gamma}\cos(\Omega_u t)$$
$$z(t) = \left(1 - \frac{1}{2\gamma^2} - \frac{K^2}{4\gamma^2} - \frac{\theta_x^{\,2}}{2}\right)ct - \frac{K^2 c}{8\gamma^2\Omega_u}\sin(2\Omega_u t) + \frac{K\theta_x c}{\gamma\Omega_u}\sin(\Omega_u t) \tag{10}$$

The electron oscillations occur at the fundamental undulator frequency in the transverse direction. An angular offset in the electron motion in the transverse direction modifies the electron longitudinal motion . The longitudinal electron velocity modulation is proportional to the amplitude and oscillation of the electron motion in the transverse direction. The terms in Eq. (6) for the second undulator section are