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A new indentation model for sandwich circular panels with gradient metallic foam cores

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ABSTRACT

A new analytical model is presented to predict indentation behavior of the sandwich circular panel with gradient foam cores under a flat-end cylindrical indenter. In the model, a displacement field of the upper face sheet of the sandwich panel is assumed to be a cosine function and plateau stress of the gradient foam core varies with the mass density along the thickness direction of the sandwich panel. The sandwich panel is modeled as an infinite, isotropic, plastic membrane on a rigid-plastic foundation. The explicit solutions of the relation between the indentation force and maximum plastic regions of the upper face sheet are derived based on the principle of minimum work. The analytical results are validated using the finite element code ABAQUS®. The influences of the gradient foam core on the maximum plastic region, the indentation force and the plastic strain energy of the sandwich panel are also investigated.

1. Introduction

Lightweight sandwich panels with metallic foam cores are being developed for structures that require high specific stiffness or strength [1–5]. The one of newest focus is gradient foam core of sandwich structures, which is a gradient pore structures from one surface of the material to the other resulting in varying material properties [6–9]. The use of gradient foam cores in sandwich structures can change structural flexural bearing capacity as required. Research results [10] show that gradient foams are superior in their energy-absorbing capability than homogeneous foams. Owing to the low strength of the core and low bending stiffness of the thin face sheets, the improvement of sandwich structures strongly depends on the indentation involved in the deformation [11]. So it is very meaningful to investigate the indentation behavior of sandwich structures with gradient foam core.

In earlier indentation analyses of sandwich structures, general analytical models focused on elastic response of the whole sandwich structures [12,13], or on elastic face sheets and plastic foam core of the sandwich structures [14,15].

Recently, Wierzbicki and Hoo [16] proposed a large deformation model for a plastic string resting on a plastic foundation subjected to a concentrated load. This model improved by a quadratic polynomial function was applied to predict the local indentation of the sandwich panel with homogenous foam core [17].

All of the literature mentioned above are concerned with sandwich panel with homogenous foam core and no one gives a solution to sandwich panel with gradient foam core. Thus it is necessary to build one model for indentation analyses of sandwich structures with gradient foam core dented by a rigid indenter.

In this study, a new displacement filed which is the cosine function instead of the quadratic polynomial function is proposed to analyze the plastic indentation response of the sandwich panel with gradient foam core. The theoretical solutions for the indentation forces and shape functions of deformed zones of the sandwich panels are derived based on the principle of minimum potential energy. An finite element model is established by using the ABAQUS code to verify the validity and applicability of the analytical solutions. The influences of the gradient metallic foam core on the maximum plastic region, the indentation force and the plastic strain energy of sandwich panels with gradient metallic foam core are also investigated.

2. Indentation model of sandwich panels with gradient foam core

Consider a sandwich circular panel with gradient metallic foam core dented by a flat-end cylindrical indenter with a radius R, as shown in Fig. 1. The sandwich circular panel is composed of face sheets and a gradient foam core with thicknesses h and c,

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respectively. The radius of the circular panel is an infinite length. The indentation depth of circular indenter is denoted by δ, while the deformation field of the face sheet in loading direction is represented by $w(r,\xi)$ with r being the horizontal coordinate. The radius of the deformed region is denoted by ξ which will be determined later.

Elastic response of the sandwich panel is neglected in present model. The upper face sheet with a flow stress σ_0 is modeled as an infinite, ideally plastic membrane resting on a rigid-plastic foundation, which gives a gradient crushing resistance by the gradient foam core.

2.1. Material properties of the gradient foam core

The gradient foam core can be considered as homogeneous metallic cellular material arranged along the thickness direction, whose initial quasi-static plateau stress linearly varies along the thickness direction. The plateau stress σ_{gc} of the gradient foam core can be calculated by

$$\sigma_{gc} = \frac{\sigma_{cb} - \sigma_{ct}}{c} z + \sigma_{ct} \tag{1}$$

where σ_{ct} and σ_{cb} are the constant plateau stress of the top surface and the bottom surface of the gradient foam core, respectively. The plateau stress σ_{gc} increases from the top surface of the core to its bottom surface as shown in Fig. 1.

2.2. Load-indentation characteristics

Based on the principle of minimum potential energy, an approximate solution of indentation response of sandwich panels can be derived. The total potential energy of the system Π is

$$\Pi = U_f + U_{gc} - W \tag{2}$$

where U_f is the plastic strain energy of the face sheet, U_{gc} the plastic work due to compressive deformation of the gradient foam core, and W the external work calculated by

$$W = \int_0^{\delta} P d\delta \tag{3}$$

A deformation field is generally assumed in these problems. Wierzbicki [16] and Xie [17] proposed and applied a quadratic polynomial displacement field to describe the deformation of upper face sheet of the sandwich panel with homogeneous foams. However, the quadratic polynomial displacement filed is too complicated to apply to indentation analyses of the sandwich panel with gradient foam core. Here, we propose an approximate shape function to describe the deformation field of the face sheet. The deformation profile is defined in terms of two independent variables: the indentation depth δ of the indenter and the radius ξ of deformation zone. The indented surface of the face sheet under a flat-end cylindrical indenter is described as follows:

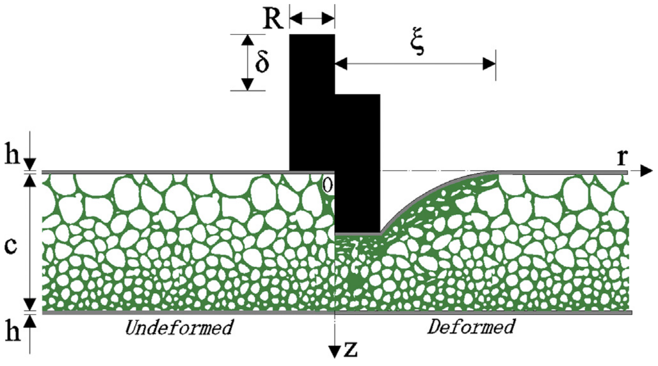

Fig. 1. Schematic profiles of undeformed and deformed zones of sandwich circular panels loading under a flat-end cylindrical indenter.

$$w(r,\xi) = \begin{cases} \delta, & 0 \le r \le R \\ 2\delta\left[1 - \cos\frac{\pi}{3}\left(1 - \frac{r-R}{\xi - R}\right)\right], & R < r \le \xi \end{cases} \tag{4}$$

And then the plastic strain energy of upper face sheet can be calculated by

$$U_f = \int_A N_0 \varepsilon_r dA \tag{5}$$

where A is the deformation area of the upper face sheet, N_0 and ε_r are fully plastic membrane force, and radial tensile strain, respectively. And they can be calculated by

$$N_0 = \sigma_0 h \tag{6}$$

$$\varepsilon_r = \frac{1}{2}\left(\frac{\partial w}{\partial r}\right)^2 \tag{7}$$

where σ_0 is the flow stress of the upper face sheet. Then substituting Eqs. (6) and (7) into Eq. (5), the expression of U_f can be obtained as follows:

$$U_f = \frac{\pi \sigma_0 h \delta^2}{\xi - R}(0.3455\xi + 0.9391R) \tag{8}$$

The plastic work due to compressive deformation of the gradient foam core is calculated by

$$U_{gc} = \int_V \sigma_{gc} dV = \int_0^{\xi} \sigma_{gc} w(r,\xi) 2\pi r dr \tag{9}$$

where V is the volume of the crushed foam and σ_{gc} is the gradient plateau stress of gradient foam core. Substituting Eqs. (1) and (4) into Eq. (9), the expression of U_{gc} can be obtained

$$\begin{aligned} U_{gc} = {} & \sigma_{ct}\delta\pi\left(0.1762\xi^2 + 0.3397\xi R + 0.4841R^2\right) \\ & + \frac{\sigma_{cb} - \sigma_{ct}}{c}\delta^2\left(0.2172\xi^2 + 0.8736\xi R + 2.0492R^2\right) \end{aligned} \tag{10}$$

Then Eqs. (3), (8) and (10) are substituted into the principle of minimum potential energy of Eq. (2), the total potential energy is determined as

$$\Pi = U_f + U_{gc} - \int_0^{\delta} P d\delta \tag{11}$$

By minimizing Π with respect to δ, the indentation force can be determined as

$$\begin{aligned} P & = \frac{\partial U_f}{\partial \delta} + \frac{\partial U_{gc}}{\partial \delta} \\ & = \frac{(\sigma_{cb} - \sigma_{ct})}{c}\delta\left(0.4344\xi^2 + 1.7472\xi R + 4.0984R^2\right) \\ & \quad + \sigma_{ct}\pi\left(0.1762\xi^2 + 0.3397\xi R + 0.4841R^2\right) \\ & \quad + \frac{2\pi\sigma_0 h\delta}{\xi - R}(0.3455\xi + 0.9391R) \end{aligned} \tag{12}$$