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Three-dimensional free vibration analysis of functionally graded annular sector plates with general boundary conditions

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ABSTRACT

The main purpose of this paper is to investigate free vibration behaviors of functionally graded sector plates with general boundary conditions in the context of three-dimensional theory of elasticity. Generally, the material properties of functionally graded sector plates are assumed to vary continuously and smoothly in thickness direction. However, the changes in the material properties may occur in the other directions, such as radial direction. Therefore, two types of functionally graded annular sector plates are considered in the paper. In this work, both the Voigt model and Mori-Tanaka scheme are adopted to evaluate the effective material properties. Each of displacements of annular sector plate, regardless of boundary conditions, is expressed as modified Fourier series which consists of three-dimensional Fourier cosine series plus several auxiliary functions introduced to overcome the discontinuity problems of the displacement and its derivatives at edges. To ensure the validity and accuracy of the method, numerous examples for isotropic and functionally graded sector plates with various boundary conditions are presented. Furthermore, new results for functionally graded sector plates with elastic restraints are given. The effects of the material profiles and boundary conditions on the free vibration of the functionally sector plates are also studied.

1. Introduction

Functionally graded materials (FGMs) belong to a new class of composite materials and their material properties vary continuously along desired directions. Such materials offer a number of advantages over laminated composite materials including improved fatigue resistance, reduction in the thermal, residual and interlaminar stresses, and more desirable joining capabilities [1]. In recent years, FGMs have been utilized to build various plate structures [2–5]. As one of common plate structures, FGM annular sector plates are extensively used in various engineering fields due to their design flexibility and high load-carrying capacity, especially in aerospace, mechanic and marine industries. These applications are often exposed to severe vibration conditions. Therefore, the accurate prediction of the dynamic behavior of such plates becomes significant for designers and engineers.

Typically, the analysis of annular sector plates is performed using two-dimensional (2-D) plate theories, such as the classical plate theory (CPT) [6–9], first-order shear deformation theory (FSDT) [10–14] and higher-order shear deformation theory (HSDT) [15]. It is should noted that those 2-D plate theories are developed on the basis of certain assumptions which simplify the formulation and solution in analytical and computational procedure, but also inherently bring out errors at the same time. As a result, in order to more accurate representation for free vibration of annular sector plates, some investigations were carried out by three-dimensional (3-D) elasticity theory [16–19]. Apart from the plate theories, a number of analytical and numerical methods have been proposed and developed to deal with the vibration problems of annular sector plates, such as finite element method [7,17], variational method [8,9], Rayleigh-Ritz method [10,16,18], differential quadrature method [12,14,19].

According to a comprehensive survey of literature, it is found that a huge amount of research effort has been devoted to analyze FGM annular sector plates [20–39]. Hosseini-Hashemi et al. [20] investigated bucking and free vibration behaviors of radially functionally graded circular and annular sector plate subjected to uniform in-plane compressive loads and resting on the Pasternak elastic foundation by means of differential quadrature method and CPT, and various combinations of simply supported and clamped boundary conditions are considered. The differential quadrature method was also used by Mirtalaie and Hajabasi [22] to study the

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free vibration of functionally graded thin annular sector plates under various boundary conditions whose material properties are assumed to vary continuously through the thickness. Baferani et al. [23] presented an exact analytical method for free vibration analysis of functionally graded thin annular sector plates resting on Winkler and Pasternak foundations. The simply supported radial edges and arbitrary conditions along the circular edges are considered. Free vibration analysis of moderately thick functionally graded conical, cylindrical shell and annular plate structures was carried out by Tornabene et al. [25] using the differential quadrature (DQ) method based on the first-order shear deformation theory in which two different power-law distributions were took into account. Saidi et al. [29] analyzed free vibration of moderately thick functionally graded annular sector plates on the basis of the first-order shear deformation plate theory, and the accurate natural frequencies of the FGM annular sector plates with simply supported radial edges were presented. Nie and Zhong [35] studied the free and forced vibration of functionally graded annular sectorial plates with simply supported radial edges and arbitrary circular edges using the combination of the state space method and differential quadrature method based on the three-dimensional theory of elasticity. Tahouneh and Yas [36,37] investigated the free vibration of thick functionally graded annular sector plates with simply supported radial edges in the context of three-dimensional theory of elasticity and a semi-analytical approach composed of the different quadrature method and series was adopted.

The above review of the literature reveals that most investigations concerning FGM annular sector plate vibrations are focused on 2-D theories and the 3-D elasticity solution for free vibration of such plates seems to be limited, and the literature on radially functionally graded is also limited. Moreover, it also should noted that most of previous research efforts were restricted to limited sets of classical boundary conditions such as free, simply supported and clamped boundary conditions. However, there exist some deviations from those ideal boundary conditions in practical engineering applications, such as elastic restraints. Consequently, the main purpose of this paper is to investigate free vibration behaviors of functionally graded sector plates with general boundary conditions in the context of three-dimensional theory of elasticity. The material properties of functionally graded sector plates are assumed to vary continuously in thickness or radial direction and two kinds of material distribution are considered. The formulation is derived by the variational principle in conjunction with modified Fourier series which consists of three-dimensional Fourier series plus several auxiliary functions introduced to overcome the discontinuity problems of the displacement and its derivatives at edges. To ensure the validity and accuracy of the method, numerous examples for isotropic and functionally graded sector plates with various boundary conditions are presented. Furthermore, new results for functionally graded sector plates with elastic restraints are given, which can be serve as benchmark solutions. The effects of the material profiles and boundary conditions on the free vibration of the functionally sector plates are also studied.

2. Theoretical formulations

2.1. Functionally graded annular sector plates

Consider a functionally graded annular sector plate with inner radius R_1, outer radius R_2, thickness h and sector angle α, as shown in Fig. 1. The cylindrical coordinate system (r, θ, z) is taken to describe the displacement components u, v and w in the radial, circumferential and thickness directions which is local on the bottom surface. The annular sector plate domain is bounded by $0 \le r \le R_2 - R_1, \quad 0 \le \theta \le \alpha, \quad 0 \le z \le h$.

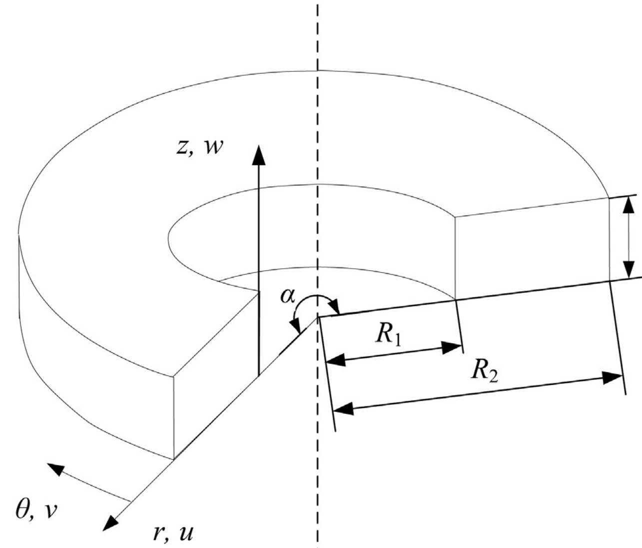

Fig. 1. The coordinate system and geometry of a thick annular sector plate.

Typically, the fabrication of FGMs can be considered by mixing two discrete phases of materials, for example, a distinct mixture of a ceramic and a metal. The effective material properties of FGMs are usually assumed to vary continuously in the thickness direction. However, the changes in the material properties may occur in the other directions, such as radial direction, when FGMs are exposed to high temperature, chemical reactions, high-level radioactivity, and so on [20,21]. In this work, the material properties of functionally graded sector plates are assumed to vary continuously in thickness or radial direction, as shown in Fig. 2. For type I the material properties vary smoothly and continuously in thickness direction; For Type II the material properties vary smoothly and continuously in radial direction.

Several models have been used over the years to estimate the effective material properties of FGMs, such as Voigt model, Mori-Tanaka scheme and the self-consistent method. The detailed descriptions on those models can be founded in monograph by Shen [40]. In this work, both the Voigt model and Mori-Tanaka scheme are adopted to evaluate the effective material properties of FGMs.

The Voigt model is simple and convenient to use for predicting the overall material properties. The effective material properties of FGMs can be expressed according to:

$$E_f = (E_1 - E_2)V_1 + E_2, \quad \rho_f = (\rho_1 - \rho_2)V_1 + \rho_2, \quad \mu_f = (\mu_1 - \mu_2)V_1 + \mu_2 \tag{1.a-c}$$

where the subscripts 1 and 2 denote the ceramic and metallic constituents, respectively. E_i, ρ_i and μ_i ($i = 1$ and 2) are the Young's modulus, mass density and Poisson ratio. V_1 is the volume fraction of ceramic, and the sum of the volume fractions of both the constituent materials makes one.

The Mori-Tanaka scheme is more accurate micromechanics model to predict the material properties of graded microstructure which have a well-defined continuous matrix and discontinuous particulate phase. It is assumed that the subscripts 1 and 2 denote the particulate and matrix phases. The effective local buck modulus K_f and the shear modulus G_f obtained by Mori-Tanaka scheme are given by

$$\frac{K_f - K_2}{K_1 - K_2} = \frac{V_1}{1 + (1 - V_1)(K_1 - K_2)/(K_2 + 4G_2/3)} \tag{2.a}$$