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Compton polarimetry revisited

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ABSTRACT

We compute the average polarisation asymmetry from the Klein–Nishina differential cross-section on free electrons at rest. As expected from the expression for the asymmetry, the average asymmetry is found to decrease like the inverse of the incident photon energy asymptotically at high energy. We then compute a simple estimator of the polarisation fraction that makes optimal use of all the kinematic information present in an event final state, by the use of "moments" method, and we compare its statistical power to that of a simple fit of the azimuthal distribution. In contrast to polarimetry with pair creation, for which we obtained an improvement by a factor of larger than two in a previous work, here for Compton scattering the improvement is only of 10–20%.

1. Cosmic-source polarimetry: the high-energy frontier

Polarimetry is a powerful diagnostic of specific phenomena at work in cosmic sources in the radio-wave and optical energy bands, but very few results are available at high photon energies: the only significant observation in the X-gamma energy range, to date, is the measurement of a linear polarisation fraction of $P = 19 \pm 1\%$ of the 2.6 keV emission of the Crab nebula by a Bragg polarimeter on board OSO-8 [1]. At higher energies, hard-X-ray and soft-gamma-ray telescopes that have flown to space in the past (COMPTEL [2], BATSE [3]) were not optimised for polarimetry, and their sensitivity to polarisation was poor. Presently active missions (Integral IBIS [4,5] and SPI [6]) have provided some improvement, with, in particular, mildly significant measurements of $P = 28 \pm 6\%$ (130 to 440 keV [6]) and $P = 47^{+19}_{-13}\%$ (200–800 keV [4]) for the Crab nebula. A number of Compton polarimeter/telescope projects have been developed, some of which also propose to record photon conversions to e^+e^- pairs. A variety of technologies have been considered, such as scintillator arrays (POGO [7], GRAPE [8], POLAR [9]), Si or Ge microstrip detectors (MEGA [10], ASTROGAM [11]) or combinations of these (Si + $LaBr_3$ for GRIPS [12], Si + CsI(Tl) for TIGRE [13]), semiconductor pixel detectors (CIPHER [14]) and liquid xenon (LXeGRIT [15]) time projection chambers (TPC). In most Compton telescopes the reconstruction of the direction of the incident photon provides an uncertainty area which has the shape of a thin cone arc. The tracking of the recoil electron from the first Compton interaction with a measurement of the direction of the recoil momentum, as is within reach with a gas TPC, allows to decrease the length of the arc and therefore to improve dramatically the sensitivity of the detector ([16] and references therein).

Some of these telescopes are sensitive to photon energies up to tens of MeV in the Compton mode, but their sensitivity to polarisation above a few MeV is either nonexistent or undocumented.

2. Polarisation asymmetry and average polarisation asymmetry

As is well known, the sensitivity to polarisation of Compton scattering is excellent at low energies (Thomson scattering), as the polarisation asymmetry $\mathcal{A}$, also known as the modulation factor and defined by the phase-space dependence of the differential cross-section

$$\frac{d\sigma}{d\Omega} \propto \left[1 + \mathcal{A}P\cos\left[2(\phi - \phi_0)\right]\right], \tag{1}$$

reaches -1 at a polar angle θ of 90° (Fig. 1 of Ref. [17]). In this expression, ϕ is the azimuthal angle, that is the angle between the scattering plane and the direction of polarisation of the incident photon. Unfortunately, $\mathcal{A}$ is decreasing with energy, and as the precision of the measurement scales as $\sigma_P \propto 1/(\mathcal{A}\sqrt{N})$ when the background noise is negligible and where N is the number of signal event, the sensitivity of Compton polarimetry decreases at high energies. With the goal of a quantitative assessment of this sensitivity, in this paper we compute the average polarisation asymmetry $\langle\mathcal{A}\rangle$ from the Klein–Nishina differential cross-section on free electrons at rest [18,19]. $\langle\mathcal{A}\rangle$ is defined from the differential

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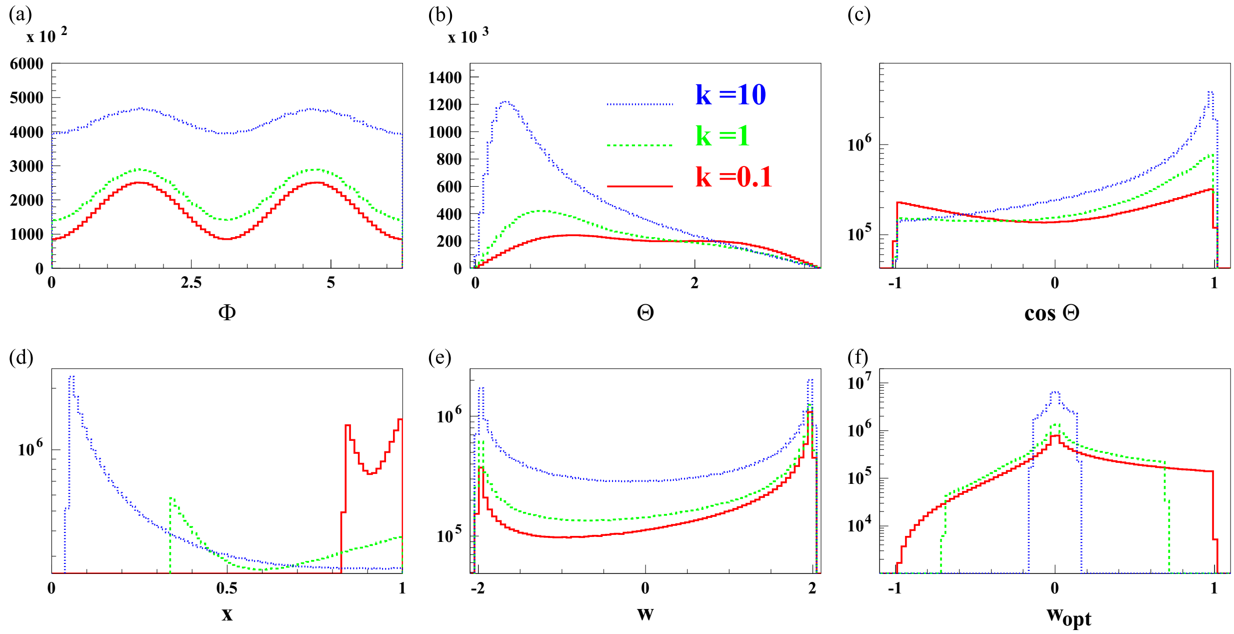

Fig. 1. Spectra of (a) and (b), the azimuthal and polar angles ϕ and θ, (c) of $\cos\theta$, (d) of the fraction x of the incident photon energy carried away by the scattered photon, and (e) and (f) of the 1D and 2D weights w and w_{opt}, for incident photon energies $0.1\,mc^2$ (continuous line), mc^2 (dashed), and $10\,mc^2$ (dotted), all for a fully polarised beam.

cross-section in ϕ, that is after the full differential cross-section Eq. (1) has been integrated over the other variables that describe the final state:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\phi} \propto \left[1 + \langle\mathcal{A}\rangle P \cos\left[2(\phi-\phi_0)\right]\right]. \tag{2}$$

Following Heitler [20], the doubly differential cross-section for linear polarised radiation reads

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{r_0^2}{2} x^2 \left[x + \frac{1}{x} - 2\sin^2\theta\,\cos^2\phi\right], \tag{3}$$

where $x = k/k_0$, k_0 and k are the energy of the incident and scattered γs, respectively; θ is the scattering angle, that is the polar angle of the direction of the scattered γ with respect to the direction of the incident γ. The differential element $\mathrm{d}\Omega$ is $\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$ as usual. In the case of partially polarised emission with polarisation fraction P, the differential cross-section becomes

$$\mathrm{d}\sigma = \frac{r_0^2}{2} x^2 \left[x + \frac{1}{x} - \sin^2\theta\left(\cos(2\phi)P + 1\right)\right] \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi. \tag{4}$$

The minus sign reflects the fact that photons Compton scatter preferentially into the direction perpendicular to the orientation of the electric field of the incoming radiation. The energy of the scattered γ is related to θ from energy-momentum conservation: $k = k_0/[1 + k_0(1-\cos\theta)]$, $x = 1/[1 + k_0(1-\cos\theta)]$, $\cos\theta = 1 - (1/x - 1)/k_0$, $\sin\theta\,\mathrm{d}\theta = -\mathrm{d}x/(k_0 x^2)$ and $\sin^2\theta = 2[1/(xk_0) - 1/k_0] - [1/(xk_0) - 1/k_0]^2$. We then obtain [20,21]

$$\mathrm{d}\sigma = \frac{r_0^2}{2k_0}\left[x + \frac{1}{x} - \left[2\left(\frac{1}{xk_0} - \frac{1}{k_0}\right) - \left(\frac{1}{xk_0} - \frac{1}{k_0}\right)^2\right]\left(\cos(2\phi)P + 1\right)\right]\mathrm{d}x\,\mathrm{d}\phi. \tag{5}$$

k varies in a range such that $-1 \le \cos\theta \le 1$, that is $1/(1+2k_0) \le x \le 1$. The distributions of these kinematic variables are shown in Fig. 1. After an elementary integration over x, we obtain

$$\mathrm{d}\sigma = r_0^2\left[\frac{1+k_0}{k_0^3}\left(\frac{2k_0(1+k_0)}{1+2k_0} - \ln(1+2k_0)\right) + \frac{\ln(1+2k_0)}{2k_0} - \frac{1+3k_0}{(1+2k_0)^2} + \frac{(2k_0-(k_0+1)\log(2k_0+1))}{k_0^3}\cos(2\phi)P\right]\mathrm{d}\phi, \tag{6}$$

that is a total cross-section of [18,20,21]

$$\sigma = 2\pi r_0^2 \times \left[\frac{1+k_0}{k_0^3}\left(\frac{2k_0(1+k_0)}{1+2k_0} - \ln(1+2k_0)\right) + \frac{\ln(1+2k_0)}{2k_0} - \frac{1+3k_0}{(1+2k_0)^2}\right]. \tag{7}$$

Equating the constant term and the term proportional to $\cos(2\phi)P$ in Eqs. (2) and (6), we obtain the average polarisation asymmetry:

$$\langle\mathcal{A}\rangle = \frac{(2k_0-(k_0+1)\log(2k_0+1))}{(1+k_0)\left(\frac{2k_0(1+k_0)}{1+2k_0} - \ln(1+2k_0)\right) + \frac{k_0^2\ln(1+2k_0)}{2} - \frac{(1+3k_0)k_0^3}{(1+2k_0)^2}}. \tag{8}$$

We now examine two limiting cases:

- At low energies, $k_0 \approx 0$, Eq. (6) reduces to

$$\mathrm{d}\sigma = \frac{r_0^2}{2}\left[1 - \cos(2\phi)P/2\right]\frac{k_0}{3}\mathrm{d}\phi, \tag{9}$$

  which results in a total cross-section of $\sigma = 8r_0^2\pi/3$, i.e., the Thomson cross-section. The low-energy average asymmetry is $\langle\mathcal{A}\rangle = -1/2$.
- At high energies,

$$\mathrm{d}\sigma = \frac{r_0^2}{2k_0}\left[\log(2k_0) + \frac{1}{2} + P\frac{4 - 2\log(2k_0)}{k_0}\cos(2\phi)\right]\mathrm{d}\phi, \tag{10}$$

  which results in a total cross-section of $\sigma = \pi r_0^2(\log(2k_0) + \frac{1}{2})/k_0$. The high-energy average asymmetry is

$$\langle\mathcal{A}\rangle = -\frac{4(\log 2k_0 - 2)}{k_0(2\log 2k_0 + 1)}. \tag{11}$$

The average asymmetry decreases at high energies, asymptotically approaching $\langle\mathcal{A}\rangle \approx -2/k_0$. The variation of the average polarisation asymmetry of photon Compton scattering on free electrons at rest (Eq. (8)) is compared to its high-energy approximation (Eq. (11)) in Fig. 2. The absence of sensitivity of Compton