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An effective thickness to estimate stresses in laminated glass beams under dynamic loadings

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Finite element models for estimating stresses and displacements in laminated glass elements under dynamic loadings are very time-consuming because (1) many small 3D elements are needed to model accurately all the layers of the sandwich element and (2) the core usually shows a time and temperature dependent behaviour. In the last years, the concept of effective thickness using a quasi-elastic solution has got the attention of the research community because of its simplicity and reasonable level of accuracy achieved in the calculation of laminated glass elements under static loadings. In this paper, a dynamic effective thickness to estimate stresses in laminated glass beams under dynamic loadings in the frequency domain is derived using the correspondence principle. The analytical equations are validated by experimental tests carried out on simply supported and free–free laminated glass beams at different temperatures in the range 20–40 °C.

1. Introduction

Laminated glass is a sandwich or layered material which consists of two or more plies of monolithic glass with one or more interlayers of a polymeric material. All polymeric interlayers present a viscoelastic behaviour, i.e. their mechanical properties are frequency (or time) and temperature dependent [1,2]. Polyvinyl butyral (PVB) is the most used interlayer material.

In analytical and numerical models, glass mechanical behaviour is usually modelled as linear-elastic in the pre glass-breakage, whereas the polymeric interlayer is characterized as linear-viscoelastic [1]. Laminated glass is easy to assemble in a finite element models but a lot of small 3D elements are needed to mesh accurately because the thickness of the viscoelastic interlayer is usually very small compared with the dimension of the laminated glass element. Consequently, the 3D models are highly time-consuming.

In the last years several analytical models have been proposed for determining the static deflections and stresses of laminated glass beams [2–8]. In order to simplify the calculation of deflections and stresses in laminated glass beams, the concept of effective thickness have been proposed in the literature [7,8]. The method consists of calculating the thickness (time and temperature dependent) of a monolithic element with bending properties equivalent to those of the laminated one, that is to say, the deflections and stresses provided by the equivalent monolithic beam are equal to those of the layered model with viscoelastic core.

With respect to the dynamic behaviour, several models were proposed in the 60s and 70s about the dynamic flexural vibration of sandwich beams with viscoelastic core [9–14]. Aenlle and Pelayo [15] demonstrated that the model of Ross, Kerwin and Ungar (RKU) [9] can be considered as a particular case of the Mead and Markus model [12] when the exponential decay rate per unit length along the beam is neglected. The authors derived an effective stiffness for the dynamic behaviour of laminated glass beams from the RKU model [9], which can be used to calculate modal parameters and dynamic deflections in laminated glass beams. With this technique, monolithic numerical models with an effective stiffness [17,18] can be used advantageously in place of layered models.

A dynamic effective thickness for laminated glass plates was proposed by Aenlle and Pelayo [16]. Furthermore, the authors proposed the effective Young modulus concept for beams and plates which is more attractive for using in numerical models. The effect of temperature in the dynamic behaviour of laminated glass elements was studied in Refs. [15,16,19].

The aim of this paper is to propose a simplified method to estimate stresses in the frequency domain in laminated glass beams subject to dynamic loadings using an equivalent monolithic model,

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Nomenclature

E	Young modulus
E_{eff}	effective Young modulus
E_1	Young's modulus of glass layer 1
E_3	Young's modulus of glass layer 3
$E_2^*(\omega)$	complex tensile modulus for the polymeric interlayer
$E_2'(\omega)$	real component of the tensile complex modulus (storage)
$E_2''(\omega)$	imaginary component of the tensile complex modulus (loss)
$E_2(t)$	viscoelastic relaxation tensile modulus for polymeric interlayer
$G_2(t)$	viscoelastic relaxation shear modulus for the polymeric interlayer
$G_2^*(\omega)$	complex shear modulus for the polymeric interlayer
$G_2'(\omega)$	real component of the shear complex modulus (storage)
$G_2''(\omega)$	imaginary component of the shear complex modulus (loss)
H_1	thickness of glass layer 1 in laminated glass
H_2	thickness of polymeric layer in laminated glass
H_3	thickness of glass layer 3 in laminated glass
$H_0 = H_2 + ((H_1 + H_3)/2)$	
I	second moment of area
$I_1 = H_1^3/12 \;\; I_3 = H_3^3/12$	
$I_T = I_1 + I_3 = (H_1^3 + H_3^3)/12$	
$K_2(t)$	viscoelastic bulk modulus
$K_2^*(\omega)$	
L	length of a glass beam
T	temperature
T_0	reference temperature
$Y = (H_0^2 E_1 H_1 E_3 H_3)/(EI_T(E_1 H_1 + E_3 H_3))$	

Lowercase letters

a_T	shift factor
b	width of a glass beam
$g(x)$	shape function (Galuppi and Royer Carfagni model)
i	imaginary unit
k_I	wavenumber
$\overline{m}$	mass per unit area
t	time
w	deflection

Greek letters

Ω^*	non-dimensional complex frequency
ζ	modal damping ratio
η	loss factor
η_2	loss factor of the polymeric interlayer of laminated glass
ρ_i	mass density of laminated glass layers
ω	frequency

avoiding the use of layered finite element models or complicated analytical models. A dynamic stress effective thickness for laminated glass beams in the frequency domain is derived by applying the correspondence principle [20–23] to the stress effective thickness for static loadings proposed by Galuppi and Royer-Carfagni [8]. The dynamic stress effective thickness is dependent on the dynamic effective stiffness proposed by Aenlle and Pelayo [15,16] to estimate modal parameters and dynamic deflections. Equations for the stress effective Young Modulus and the stress effective distance to the neutral axis are also formulated which can be used in place of the effective thickness with the same accuracy. This technique can be applied to three layered laminated glass beams with glass showing a linear elastic behaviour and the polymeric core showing viscoelastic behaviour [1,20–23]. In order to validate the model, the stresses in a laminated glass beam made of annealed glass plies and PVB core were estimated using the stress effective thickness concept. The analytical predictions were validated with experimental tests comparing the predicted stresses with those measured with strain gages.

1.1. Viscoelastic behaviour

The mechanical properties of a linear-viscoelastic material are frequency (or time) and temperature dependent [1,22]. In the frequency domain, the complex tensile modulus, $E_2^*(\omega)$, at temperature T is given by:

$$E_2^*(\omega,T) = E_2'(\omega,T) + i\cdot E_2''(\omega,T) = E_2'(\omega,T)(1 + i\cdot\eta_2(\omega,T)) \quad (1)$$

where superscript '*' indicates complex, ω represents the frequency, i is the imaginary unit, $E_2'(\omega,T)$ and $E_2''(\omega,T)$ are the storage and the loss tensile moduli, respectively, and

$$\eta_2(\omega) = \frac{E_2''(\omega,T)}{E_2'(\omega,T)} \quad (2)$$

is the loss factor that relates both moduli. The subscript '2' is used hereafter to reference the viscoelastic interlayer (Fig. 1).

As regards the shear behaviour, the complex shear modulus, $G_2^*(\omega,T)$, is given by:

$$G_2^*(\omega,T) = G_2'(\omega,T) + i\cdot G_2''(\omega,T) = G_2'(\omega,T)(1 + i\cdot\eta_2(\omega,T)) \quad (3)$$

where $G'(\omega,T)$ and $G''(\omega,T)$ are the storage and the loss shear moduli, respectively.

Both the shear and tensile moduli can be related by means of the correspondence principle [20–23] introducing the corresponding complex viscoelastic properties, i.e.:

$$G_2^*(\omega,T) = \frac{3E_2^*(\omega,T)K_2^*(\omega,T)}{9K_2^*(\omega,T) - E_2^*(\omega,T)} \quad (4)$$

where $K_2^*(\omega,T)$ is the complex bulk modulus.

In order to take into account the temperature dependence of the viscoelastic interlayer properties, it is commonly assumed a simply thermo-rheological behaviour in the material [22]. This fact allows determining a relation between time and temperature in linear viscoelastic materials using a Time–Temperature–Superposition (TTS) model such as the William–Landel–Ferry or Arrhenius equations [1,24]. Once the TTS model is fitted for a reference temperature, T_0, i.e. the temperature used in the experimental tests, the moduli for the material to a different temperature, T_1, can be estimated by shifting in time the moduli at temperature T_0 using a shift