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Analytical benchmarks for precision particle tracking in electric and magnetic rings

E.M. Metodiev [a,c,d,f], I.M. D'Silva [a], M. Fandaros [a], M. Gaisser [d,f], S. Hacıömeroğlu [a,d,e,f], D. Huang [a], K.L. Huang [a,c], A. Patil [a], R. Prodromou [a], O.A. Semertzidis [a], D. Sharma [a], A.N. Stamatakis [a], Y.F. Orlov [b], Y.K. Semertzidis [a,d,f,*]

[a] Brookhaven National Laboratory, Physics Department, Upton, NY 11973, USA
[b] Department of Physics, Cornell University, Ithaca, NY, USA
[c] Harvard College, Harvard University, Cambridge, MA 02138, USA
[d] Center for Axion and Precision Physics Research, IBS, Daejeon 305-701, Republic of Korea
[e] Istanbul Technical University, Istanbul 34469, Turkey
[f] Department of Physics, KAIST, Daejeon 305-701, Republic of Korea

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ABSTRACT

To determine the accuracy of tracking programs for precision storage ring experiments, analytical estimates of particle and spin dynamics in electric and magnetic rings were developed and compared to the numerical results of a tracking program based on Runge–Kutta/Predictor–Corrector integration. Initial discrepancies in the comparisons indicated the need to improve several of the analytical estimates. In the end, this rather slow program passed all benchmarks, often agreeing with the analytical estimates to the part-per-billion level. Thus, it can in turn be used to benchmark faster tracking programs for accuracy.

1. Introduction

Analytical estimates for particle dynamics in electric and magnetic rings with and without focusing have been given in a variety of papers and notes. We suggest that these high-precision estimates can serve as benchmarks to test the accuracy of any precision particle tracking program. A program that successfully passes all benchmarks can, in turn, provide a baseline to benchmark faster programs. Thus, it can be a powerful tool for assessing tracking programs for Muon (g-2), Storage Ring EDM and other precision physics experiments requiring high-precision beam and spin dynamics simulation. The program we put to the test in this paper is based on Runge–Kutta/Predictor–Corrector (RKPC) integration, a relatively slow but simple method. It should reproduce the analytical estimates to sub-ppm accuracy on a time scale on the order of hours, in order to be a feasible candidate for benchmarking faster programs. We use the term "focusing" to denote "weak vertical focusing" unless otherwise indicated. Horizontal focusing is defined by the vertical focusing plus the geometry of the ring, always conforming with Maxwell's equations. These benchmarks include the following:

- Pitch correction [1,2] to particle precession frequency in a uniform B-field with and without focusing.
- Vertical oscillations and energy oscillations in a uniform B-field with no focusing, electric focusing, and magnetic focusing.
- Radial and vertical oscillations and energy oscillations in an all-electric ring with and without weak focusing.
- Synchrotron oscillations and momentum capture with a radio frequency cavity (RF) in a uniform B-field.
- An EDM signal and systematic error with an RF Wien Filter in a magnetic ring.

In the analytical estimates that follow, we define γ_0 as the Lorentz factor of the design particle in the ring. The vertical pitch angle θ_y of a particle is defined such that $\theta_y = \beta_z/\beta_\theta$ where $(\beta_\rho, \beta_\theta, \beta_z) = \vec{v}/c$ in cylindrical coordinates. The field focusing index is n, with $n = -(dB/B_0)/(dr/r_0)$ a number with range $0 < n < 1$.

2. Motivation

A tracking program to be used for estimates and investigations in precision experiments must be optimized to be as accurate and

* Corresponding author.

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fast as possible. This calls for a well-tested and robust procedure to benchmark the accuracy of tracking programs in situations relevant to the experiments. Precision experiments such as the Muon (g-2) and Storage Ring Electric Dipole Moment (EDM) experiment [3–5] require measurements of sub-part per million (ppm) accuracy. In the case of a proton or deuteron Storage Ring EDM experiment, a tracking program of extraordinary precision is required to estimate the spin coherence time of the particle distribution and various lattice parameters, as well as to estimate the values of systematic errors associated with the experiment. Many commonly used beam and spin dynamics programs ignore, or erroneously account for, second and higher-order effects. Tracking in an electric storage ring poses the additional challenge of conforming with total-energy conservation while accounting for higher-order effects.

Numerical integration with a sufficiently small step size allowed to run for a sufficiently long time may reproduce the analytical results with high accuracy. Moreover, comparison of analytical estimates with precision tracking results can identify discrepancies and indicate the need to improve the estimates. (In this way, it was determined that the total correction due to vertical particle oscillations, the so-called pitch effect, can be significantly reduced [6].)

We benchmarked a program based on Runge–Kutta/Predictor–Corrector method [7] against the developed analytical estimates.

3. Precision tracking

For a particle of mass m and charge e, there are two differential equations that govern particle and spin dynamics. For particle velocity $\vec{\beta}$ and rest spin $\vec{s}$ in external fields, the equations are [8]

$$\frac{d\vec{\beta}}{dt}=\frac{e}{m\gamma c}\left[\vec{E}+c\vec{\beta}\times\vec{B}-\vec{\beta}(\vec{\beta}\cdot\vec{E})\right], \tag{1}$$

and the T-BMT equation, with an anomalous magnetic moment a of the particle:

$$\frac{d\vec{s}}{dt}=\frac{e}{m}\vec{s}\times\left[\left(a+\frac{1}{\gamma}\right)\vec{B}-\frac{a\gamma}{\gamma+1}\vec{\beta}(\vec{\beta}\cdot\vec{B})-\left(a+\frac{1}{\gamma+1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]. \tag{2}$$

The RKPC integration was used with a step size of 1–10 ps to numerically solve the two differential equations with the corresponding initial conditions.

4. Magnetic ring

A magnetic ring consists of a uniform magnetic field $\vec{B}$, taken to be in the vertical direction. The correction C to the precession frequency due to a vertical pitch is defined by $\omega_m=\omega_a(1-C)$, where ω_a is the $(g-2)$ correct frequency [9] for a particle with anomalous magnetic moment a, and ω_m is the measured frequency. The predicted correction is [2]

$$C=\frac{1}{4}\theta_0^2\left\{1-(\omega_a^2+2a\gamma^2\omega_p^2)/\gamma^2(\omega_a^2-\omega_p^2)\right\}. \tag{3}$$

with $\omega_p=2\pi f_p$, where f_p is the vertical (pitch) oscillation frequency.

4.1. No focusing

When there is no focusing or when $\omega_p \ll \omega_a$, the correction from Eq. (3) becomes

$$C=\frac{1}{4}\beta^2\theta_0^2, \tag{4}$$

where for linear oscillations, $\langle\theta_y^2\rangle=(1/2)\theta_0^2$, where θ_0 is the maximum pitch angle of the particle trajectory.

For a particle with $\beta=0.972$ and a constant 1.0 mrad vertical pitch as shown in Fig. 1, the simulated correction to the $(g-2)$ precession frequency of 0.2361 ppm is in very good agreement with the analytically predicted value of 0.2363 ppm using Eq. 4.

Checking over several values of θ_y confirms that the analytic expression and the pitch correction in the tracking simulation agree for small θ_y, as expected.

4.2. Weak magnetic focusing

When there is magnetic focusing and when $\omega_p \gg \omega_a$, the correction from Eq. (3) becomes

$$C=\frac{1}{4}\theta_0^2(1+2a). \tag{5}$$

The analytical estimate [10] for the average particle radial deviation from the ideal orbit with radius r_0, with weak magnetic focusing index n, takes the form

$$\left\langle\frac{\Delta r}{r_0}\right\rangle=\alpha_p\left\langle\frac{\Delta p}{p_0}\right\rangle=-\frac{1}{1-n}\langle\theta_y^2\rangle, \tag{6}$$

for a vertical pitch frequency significantly greater than the $(g-2)$ precession frequency of the particle, where α_p is the momentum compaction factor.

Eq. (6) predicts an average radial deviation $\langle\Delta r/r_0\rangle$ of -5×10^{-7} using $\theta_0=$ 1 mrad and a field index $n=0.01$, consistent with the tracking results shown in Fig. 2 to sub-part per billion (ppb) level. The dependence of $<\Delta r/r_0>$ on the field index is shown to hold over a range of n values in Fig. 3.

In a continuous storage ring with weak focusing, field strength B_0, and ring radius r_0, the vertical and horizontal magnetic field components around the ideal trajectory can be expressed to second-order in the vertical position y as

$$B_x(x,y)=-n\frac{B_0}{r_0}y \tag{7}$$

$$B_y(x,y)=B_0-n\frac{B_0}{r_0}x+n\frac{B_0}{r_0}\frac{y^2}{2r_0}, \tag{8}$$

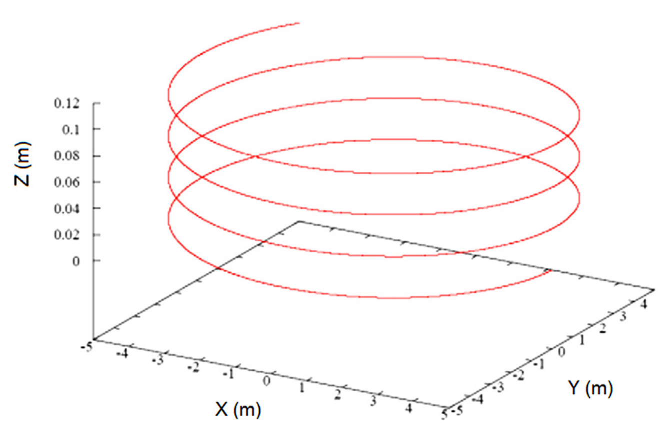

Fig. 1. The particle path in Cartesian coordinates in a uniform B-field with pitch angle $\theta_y=1.0$ mrad, for a ring with a 5 m radius.