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On the interpretation of current–voltage curves in ionization chambers using the exact solution of the Thomson problem

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ABSTRACT

The $I-\Delta V$ characteristic curve of a well type ionization chamber irradiated with ^{192}Ir sources (0.75 Ci–120 Ci) was fitted using the exact solution of the Thomson problem. The recombination coefficient and saturation current were estimated using this new approach. The saturation current was compared with the results of the conventional method based on Boag–Wilson formula. It was verified that differences larger than 1% between both methods only occurred at activities higher than 55 Ci. We concluded that this new approach is recommended for a more accurate estimate of the saturation current when it is not possible to measure currents satisfying the condition $I/I_{sat} > 0.95$. From the calibration curve the average value of pairs of carriers created per unit volume was estimated to be equal to $\eta = 8.1 \times 10^{-3}$ cm^{-3} s^{-1} Bq^{-1} and from that value it was estimated that ~ 17 pairs were created on average per second for each decay of the source.

1. Introduction

The description of the saturation characteristics of ionization chambers operating in current mode is a very old problem in physics which dates back to 1899, when J.J. Thomson set up the general differential equation for the transport of ions between parallel plates [1]. Many authors (Mie [2], Seeliger [3], Boag and Wilson [4]), including Thomson [1], proposed approximate solutions to this problem, but it was only many years later, in 1975, that a general solution was found by Rosen and George [5] with the only additional approximation of neglecting space charges. This was the first solution that linked the current induced by the motion of charges, I, with the voltage applied to the ionization chamber, ΔV, and provided an expression to the spacial distribution of charges. Nonetheless, the relation between current and applied voltage was found as an implicit formula of both variables (*i.e.* $F(I, \Delta V) = 0$), which makes the procedure of data fitting by the least squares method infeasible. Recently, Chabod [6] found a way to write ΔV explicitly as a function of I (*i.e.* $\Delta V = f(I)$). With this last achievement it is now possible to fit the experimental current–voltage curve of an ionization chamber using the exact solution to the Thomson problem as long as some theoretical assumptions are met. Furthermore, as far as the authors are concerned, there is still no published experimental work that applies this new approach in the analysis of current–voltage curves.

In this work, the experimental current–voltage (I–ΔV) curves of a well type ionization chamber irradiated with very intense ^{192}Ir sources with activities ranging from few curies to approximately one hundred curie were fitted using the analytical formula proposed by Chabod. This formula can be translated into a fitting function with only two parameters, which can be related with all the relevant physical variables of the problem: the saturation current I_{sat}, the recombination coefficient k, the rate of electron–ion pair formation per unit volume N, the electron and ion mobilities μ_e and μ_a, respectively, and the electrodes separation distance d. As previously stressed by Chabod, this fitting procedure paves the way for many interesting applications, such as the measurement of the recombination coefficient k, the determination of the fraction of charges that escape recombination, the estimate of the ionization chamber efficiency and the extrapolation of the saturation current value using parts of saturation curves.

A typical problem in the dosimetry of very intense radioactive sources is the determination of the saturation current required in calibration procedures of ionization chambers used as dose and activity meters. In these cases, very high applied voltages are

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needed to achieve the saturation plateau. In practice, such high voltages can produce sparks and hence should be avoided. In this case, the saturation current must be extrapolated from the recombination region of the I–ΔV curve. Old theories addressed this problem, but they were valid only when $I/I_{sat} > 95\%$. In many cases, the measured I–ΔV curve does not satisfy this condition and only a valid fit of the recombination region would give a good estimate of the saturation current. As an example of application, it will be shown how a calibration procedure that uses the traditional extrapolation approach (*e.g.* Boag–Wilson formula [4]) to determine the saturation current differs from a calibration procedure that uses the exact solution. Using this method, it can be shown that the upper limit of the activity range of ionization chambers may be extended to higher values.

1.1. The Thomson problem

The Thomson problem is basically the set of differential equations that describe macroscopically the transport of positive and negative charge carriers through a gaseous medium subject to a static electric field. It is assumed that: (i) the diffusion contribution to the charge velocities is negligible in comparison with the electric field contribution; (ii) there is no charge multiplication by electron impact ionization or any other ionization process; (iii) there is only two types of charge carriers: one of them is negative and is always an electron (n_e designates the electron density); the other one is positive and is always a singly ionized atom (n_a designates the positive ion density); (iv) the charge recombination occurs by a mechanism whose rate may be written as kn_an_e; (v) the pair creation is caused only by the ionizing radiation and its rate of formation per unit volume, N, is considered to be constant all over the active volume of the detector chamber; (vi) the charge velocities are proportional to the magnitude of the field, the proportionality constants being the mobilities μ_e and μ_a. Considering that the problem has planar symmetry (cf. Fig. 1) and that all the above-mentioned hypotheses are valid, plus the proper boundary conditions, the physical variables n_e, n_a and E can be coupled through the continuity equations of the positive and negative carriers and the Poisson equation

$$\begin{cases} -\mu_e \dfrac{\partial}{\partial z}(n_e E) = N - kn_e n_a \\ \mu_a \dfrac{\partial}{\partial z}(n_a E) = N - kn_e n_a \\ \dfrac{\partial}{\partial z} E = \dfrac{e}{\varepsilon_0}(n_a - n_e) \\ n_e(d) = 0, \quad n_a(0) = 0, \quad \int_0^d E(z)\,\mathrm{d}z = \Delta V \end{cases} \tag{1}$$

where e is the elementary charge and ε_0 is the vacuum permittivity. Although the above set of differential equations is apparently simple, an analytical solution to it has not been found yet.

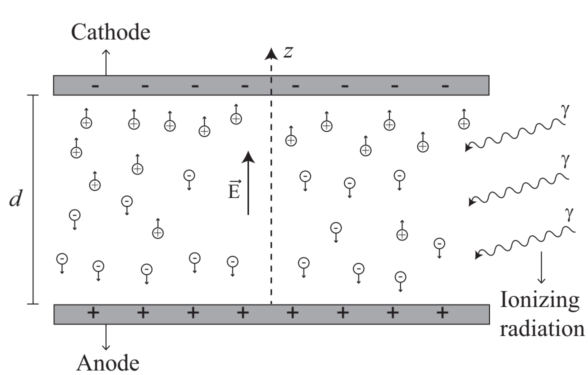

Fig. 1. Schematic view of a parallel plate ionization chamber.

Fortunately, the space charge effect may be neglected in many practical situations (*i.e.* the component of the field generated by the charged electrodes prevails over the field generated by the free carriers), and the set of equations may be simplified to

$$\begin{cases} -\mu_e E \dfrac{\partial}{\partial z} n_e = N - kn_e n_a \\ \mu_a E \dfrac{\partial}{\partial z} n_a = N - kn_e n_a \\ n_e(d) = 0, \quad n_a(0) = 0, \quad E = \frac{\Delta V}{d}. \end{cases} \tag{2}$$

This equation may be solved analytically to give the spacial distribution of negative and positive charges, as shown by Rosen and George [5]. The spacial distribution of electrons, for instance, is given by the following formula, which will be useful later when computing the space charge effects [6]

$$n_e(x) = \frac{kI - K\ \tan\left((K/2eS\nu_e\nu_a)(x+C)\right)}{2eSk\nu_e} \tag{3}$$

where S is the electrode surface area, $\nu_e = \mu_e E$ and $\nu_a = \mu_a E$ are the electron and positive ion drift velocities, respectively, and C is an integration constant which may be evaluated using the boundary condition $n_e(d) = 0$ and

$$K = \sqrt{4kNe^2S^2\nu_e\nu_a - k^2I^2}. \tag{4}$$

As it was shown by Chabod [6], the following law links the voltage ΔV to the current I:

$$\Delta V = \begin{cases} \Delta V_0 \times \dfrac{\sqrt{1+\sqrt{1-\eta^2\Xi(\eta)^2}}}{\Xi(\eta)} & \text{when } \eta \geq \beta, \\ \Delta V_0 \times \dfrac{\sqrt{1-\sqrt{1-\eta^2\Xi(\eta)^2}}}{\Xi(\eta)} & \text{when } \eta \leq \beta. \end{cases} \tag{5}$$

where $\beta = 2/\pi$ and

$$\begin{cases} \eta = \dfrac{I}{I_{sat}} \\ \Delta V_0 = \dfrac{d^2}{\sqrt{2}}\sqrt{\dfrac{kN}{\mu_e\mu_a}} \end{cases} \tag{6}$$

and the function $\Xi : [0,1] \to [0,\pi]$ is given by

$$\Xi\left(\frac{\sin(x)}{x}\right) = x. \tag{7}$$

No additional assumption is made as (5) may be derived only by simplification and rearrangement of the expression of the electron spacial distribution [6]. As it will be shown later, expression (5) may be set as the model function in the least squares procedure. In general, only two variables must be assigned as adjustable parameters: I_{sat} and ΔV_0.

The main goal of this work is to show that Eq. (5) can provide a good description of a real current–voltage curve as long as some assumptions are met. It will be shown in the next section that the experimental conditions are to a good degree of accuracy consistent with the theoretical hypotheses. By no coincidence the experimental data can be quite accurately fitted using (5); from the adjusted parameters valuable information may be extracted which, as mentioned before, relates with physical quantities and may be useful in many applications.