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Campbelling-type theory of fission chamber signals generated by neutron chains in a multiplying medium

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ABSTRACT

The signals of fission chambers are usually evaluated with the help of the co-called Campbelling techniques. These are based on the Campbell theorem, which states that if the primary incoming events, generating the detector pulses, are independent, then relationships exist between the moments of various orders of the signal in the current mode. This gives the possibility to determine the mean value of the intensity of the detection events, which is proportional to the static flux, from the higher moments of the detector current, which has certain advantages.

However, the main application area of fission chambers is measurements in power reactors where, as is well known, the individual detection events are not independent, due to the branching character of the neutron chains (neutron multiplication). Therefore it is of interest to extend the Campbelling-type theory for the case of correlated neutron events. Such a theory could address two questions: partly, to investigate the bias when the traditional Campbell techniques are used for correlated incoming events; and partly, to see whether the correlation properties of the detection events, which carry information on the multiplying medium, could be extracted from the measurements. This paper is devoted to the investigation of these questions. The results show that there is a potential possibility to extract the same information from fission chamber signals in the current mode as with the Rossi- or Feynman-alpha methods, or from coincidence and multiplicity measurements, which so far have required detectors working in the pulse mode. It is also shown that application of the standard Campbelling techniques to neutron detection in multiplying systems does not lead to an error for estimating the stationary flux as long as the detector is calibrated in in situ measurements.

1. Introduction

Fission chambers used as neutron detectors have the advantage that they can be operated both in pulse counting mode and in current mode. At low neutron intensities, the pulses, i.e. the individual current (or voltage) signals, are separated, whereas at increasing neutron intensities the pulses tend to overlap and finally form a randomly fluctuating continuous current.

The pulse counting technique, which can be performed also by many other detector types in addition to fission chambers, is capable not only to determine first order quantities, such as the average rate of incidence of the neutrons, but also the higher order moments of the process, even if the detection events are not independent. It is just the non-trivial properties of the higher order moments, such as the deviation of the variance to mean of the number or counts from unity, which are used to characterise the correlations between counting events. Namely, these carry information about the multiplication (branching) which generates the correlations between the detection events in a reactor, or on the spontaneous fission neutron source in safeguards measurements. This is the basis of the determination of subcritical reactivity in source driven systems with the Feynman- and Rossi-alpha methods, or of the coincidence and multiplicity measurements in nuclear safeguards, to characterise special nuclear materials, respectively (for a review of these methods, see [1]).

However, these methods can only be used for detection intensities sufficiently low to avoid pulse overlapping and pile-up, as well as count losses due to dead-time effects. The counting process (the hardware of the detection of level crossing) induces a non-paralysing dead time, whereas the pulse overlapping and pile-up introduces a paralysing dead-time. At high count rates leading to continuous pulse overlapping, the pulse counting technique breaks down completely.

When a fission chamber operates in the Campbelling mode, such as in a high power reactor, the advantage of the capability of determining correlations in the detection events is lost. In most cases this is not a disadvantage, since during reactor operation at full power, the main quantity of interest is the mean value (expectation) of the neutron flux. The expectation of the detector signal (its direct current), determined in practice by time averaging methods, is proportional to the neutron flux, which is the most straightforward

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way of measuring it. As mentioned before, due to the nature of the data acquisition, there are no dead-time problems present in this procedure. This of course does not mean the total absence of deviations from an idealised behaviour of the measurement system, such as space charge effects, leakage current in cables, non-negligible gamma contribution, digitising errors, etc., but it nevertheless constitutes a definite advantage.

However, in practice the procedure is used in a slightly different form. Namely, the intensity of the primary incoming events is not estimated from the expectation of the signal, rather from its higher order moments, most often from its variance. This is based on the Campbell theorem [2–4], which states that if the primary incoming events (neutron detections) are independent, the higher order moments of the detector current are also proportional to the intensity of the events (i.e. to the neutron flux), although with a different proportionality factor. This lends the possibility to determine the mean neutron flux from higher order moments of the detector signal, which has the advantage of suppressing the contribution of the gamma background.

The Campbell theorem can be stated as follows. Assuming a stationary random process $\boldsymbol{\eta}^{(st)}(t)$ representing the fluctuating detector signal, which consists of a random sum of deterministic current signals with shape $f(t)$, created according to a homogeneous Poisson process with intensity s_0 (the average rate of neutron reactions in the detector), the Campbell theorems have the form

$$\mathbf{E}\{\boldsymbol{\eta}^{(st)}(t)\} = s_0 \int_{-\infty}^{+\infty} f(t)\,dt \quad \text{and} \quad \mathbf{D}^2\{\boldsymbol{\eta}^{(st)}(t)\} = s_0 \int_{-\infty}^{+\infty} f(t)^2\,dt. \tag{1}$$

These relationships, and their higher order alternatives (relating the higher order moments to the first moment) are independent of whether the pulses are overlapping or not. Hence they create a bridge between the pulse and current modes of operation. In addition, since there is no level crossing procedure involved, only a high frequency sampling of the continuous signal is required, there are no dead-time problems involved. On the other hand, they are only valid if the incoming events are independent, hence in the classical Campbell relationships, the higher order moments do not carry any more information beyond that already contained in the first moment.

This fact raises two questions. The first concerns the recognition that fission chambers in a reactor detect primary incoming events, which are not independent, but which in the traditional applications are evaluated with the Campbell formulae that are only valid for independent incoming events. It is then of interest to explore what the consequences of this procedure are, i.e. how much bias this procedure incurs when estimating the stationary neutron flux. The second question is whether one could combine the capabilities of pulse counting techniques to explore the statistical properties of the non-independent primary events, with the advantages of the Campbell techniques, since these latter do not suffer from dead-time effects and are also applicable for strongly overlapping pulse sequences. The goal is to see whether with a form of the Campbell relationships which correctly accounts for the statistics of the incoming primary events, some alternatives of the Feynman- or Rossi-alpha methods could be developed in the current mode of the detectors. This question has recently been posed and the possibility started to get explored, both for stationary signals [5] and for pulsed neutron experiments [6].

The present paper addresses these two questions through a rigorous analytical treatment by deriving Campbell-like relationships for a detector signal, where the assumption of the independence of the primary incoming events is abandoned. This requires a suitable theoretical formalism, as well as specification of the statistical properties of the primary events. Regarding the formalism, the usual derivation of the traditional and higher order Campbell relationships is either heuristic [2–4], or rather involved [7,8]. Recently a rather transparent and effective formalism was suggested by the present authors by using the backward master equations [9,10]. The formalism can easily be extended from the case of independent incoming events to the case when the incoming neutrons were generated in a subcritical multiplying medium driven with an external source. This is made by combining the backward master equation for the detector signal, proposed in [10], with the backward master equations of neutron branching in a subcritical multiplying medium [1]. This will ensure that the temporal correlations between the detection events will correspond to a measurement in a reactor core during start-up, or in a safeguards measurement in e.g. a fuel pool, where the extraneous neutron source is the intrinsic spontaneous fission events. The resulting formulas will make it possible to investigate both the bias of the traditional Campbelling techniques when applied in measurements in multiplying systems, as well as the possibility of unfolding the parameters of the multiplying medium from the detector signal.

We shall start with the basic equations for the probability distributions and a subsequent general derivation of the moment equations. Then the expectation and the variance of the detector signal is derived in terms of the parameters of the multiplying medium and the detector pulse shape and amplitude distribution. Particular examples are shown by selecting a concrete detector pulse shape. Finally the temporal auto-covariance of the detector signal is calculated, which is an analogue of the Rossi-alpha formula of the pulse counting technique. This will make it possible to investigate the possibilities of unfolding the prompt neutron decay constant from the auto-covariance and to assess the bias of the traditional Campbell technique when applied to measurements in multiplying media. It will be shown that the prompt neutron decay constant can be determined from measurement of the auto-covariance if the detector time constant is known. It will be also seen that the traditional Campbell formulae can be used to evaluate reactor measurements as long as the detector is properly calibrated with real measurement data, but with a warning that in subcritical reactors the calibration factor depends on the subcriticality of the system.

2. Theoretical considerations – signal statistics for one source event

The goal is to determine the probability distribution of the random process $\boldsymbol{\eta}(t)$, represented by the detector signal (detector current). To this end we will use an extended version of the formalism applied in [10] to account for the statistics of the incoming particles.

Although the signal of the detector at any given time is the sum of the signals of all previous random detections, the starting point is, just as in [10], to specify the properties of the detector signal after the *detection* of *one neutron*. In [10], from these, it was immediately possible to write down a backward master equation for the probability distribution of the detector signals, due to the independence of the detection events. In the present case, it will not be possible to directly specify the probability of the arrival of the neutrons at the detector; we can only specify the probability (intensity) of the source neutron emission and hence write down a master equation for the probability distribution of the detector signal for *one source neutron*. After that, as is usual in the backward theory of branching processes, another master equation needs to be written down to connect the detector signal distribution due to one single starting neutron, with that induced by a continuous source of neutrons, which induces the stationary neutron distribution in the system (and hence the stationary distribution of the detection events).

Let $\varphi(\boldsymbol{\xi}, t)$ denote the detector signal which exists in the detector at time $t \geq 0$ after the arrival of one neutron at $t=0$. It will be assumed that this signal depends also on random parameters, which is indicated by the random variable $\boldsymbol{\xi}$ as the argument of φ.[1]

[1] In our previous work [10] it was assumed that $\varphi(\boldsymbol{\xi}, t) = \boldsymbol{\xi} f(t)$, where $f(t)$ was a deterministic, decreasing function of its argument, whereas $\boldsymbol{\xi}$ was the random