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Variance estimation in neutron coincidence counting using the bootstrap method

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ABSTRACT

In the study, we demonstrate the implementation of the "bootstrap" method for a reliable estimation of the statistical error in Neutron Multiplicity Counting (NMC) on plutonium samples.

The "bootstrap" method estimates the variance of a measurement through a re-sampling process, in which a large number of pseudo-samples are generated, from which the so-called bootstrap distribution is generated. The outline of the present study is to give a full description of the bootstrapping procedure, and to validate, through experimental results, the reliability of the estimated variance. Results indicate both a very good agreement between the measured variance and the variance obtained through the bootstrap method, and a robustness of the method with respect to the duration of the measurement and the bootstrap parameters.

1. Preliminaries

1.1. Introduction

Fissile mass estimation using passive neutron interrogation relies on counting neutrons arriving from a sample containing a spontaneous fissile material to a set of (typically ^{3}He) detectors. There is growing interest in passive neutron interrogation, due to its high performance on measuring samples containing ^{240}Pu, such as MOX fuel. To separate between the main fission source and additional neutron sources (mainly (α, n) reactions and induced fissions in fissionable isotopes in the sample), higher moments of the detected neutrons are analyzed, making use of the fact that the different neutron sources have a very different statistical nature. Such general considerations are known as neutron multiplicity counting (NMC, also referred to as time correlation analysis – TCA, or coincidence counting), and are considered a standard procedure in both the safeguards and safety communities. Because implementation of NMC methods requires sampling the third moment of a distribution, a straightforward estimation of the variance requires sampling the 6th moment of the distribution, suggesting a very high uncertainty.

In the present study, we further develop the application of the bootstrap method to estimate the random variance in mass estimation in NMC measurements. The idea of implementing the bootstrapping method on NMC was (to the best of our knowledge) originally introduced in [1] and further developed [2]. The bootstrap was implemented on actual measured data in [3], but on a very limited data set (a single measurement). In both [1] and [2] the main emphasis was not on the bootstrap; in [1] the main effort was on analytic expressions for the second and third reduced sample moments, and [2] was a more general discussion on the possible advantages of the LIST-mode data acquisition machinery (rather than the traditional shift register machinery), and many important points regarding the bootstrap method were not discussed. In particular, the aim of the present study is to further extend the work introduced in [1,2] in two aspects:

1. A more detailed definition of the bootstrapping procedure; in particular, the characteristic time scale of the shuffled intervals, and the number of re-samples needed will be discussed.
2. Validation of the method through experimental results.

The paper is arranged in the following manner: In the remaining of the present section we give the motivation for this study. In Section 2, we give a general introduction to the bootstrap method. In Section 3 we discuss the implementation of the bootstrap method on NMC. Section 4 is devoted to experimental results, and in Section 5 we conclude.

1.2. Motivation

Estimating the random variance of in NMC measurements is of utmost importance in estimating the reliability of a detection

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system. Naturally, the measurement variance has been studied before, and there are several publications regarding both the estimation [1,6,7] and the optimization [8,9] of the measurement variance. The emphasis in previous works is mainly on analytic expressions for the random variance, and the bootstrap method (or any other re-sampling method) was somewhat neglected. On the other hand, we find that the bootstrapping method has several appealing features:

1. *Simplicity*: the method, as we demonstrate, is very straightforward. The implementation is simple and does not demand any special apparatus or any complex computations. The only demand is the data that is recorded using a LIST-mode acquisition machinery [11].
2. *Robustness*: The random variance of the mass estimation depends on several parameters, depending on the measurement system (such as the detection efficiency), the tested sample (mass and leakage multiplication) and user defined parameters (gate width and measurement time). The present method is, in a sense, "blind" to all these parameters. The random variance is estimated directly through the bootstrap distribution of the mass, and no calibration is needed. It should be clear, however, that *systematic* errors related to the different parameters *are not* estimated using this method.
3. *Applicability for all NMC methods*: In addition to the well-studied multiplicity method [10], in the recent years two new methods were introduced: the random trigger interval (RTI) [12] and the skewness-variance-mean (SVM) [13]. In the present study, although the bootstrap distribution is evaluated for the *mass*, the data manipulation is done directly on the detection signal, regardless of the method used to evaluate the mass. Therefore, the method can be applied to all three methods (or any future method).

It should be stated that the bootstrap has one inherent flaw: Error approximation can only be done after the measurement is done. Thus, it is hard to pre-determine the measurement duration needed to obtain a certain statistical error using the bootstrap method. As stated, from a technical point of view, implementation of the method presented in this study has a single requirement: data recording must be through a LIST-mode data acquisition machinery, recording the time stamps of all the neutron detections.

2. Bootstrapping: the basic idea

The boot strapping procedure, introduced in [14], is aimed to determine the statistical variance of any estimator of the sampled data, by means of re-sampling from the data itself. To properly understand the bootstrap method, we need to define two basic notations:

- *The original sample* – the database, obtained from the measurement, that is available for statistical analysis.
- *The original population* – the data set from which the original sample was drawn.

Ideally, statistical uncertainty could be calculated by drawing many equally sized samples from the original population. This is of course not practical – statistical uncertainty estimation should be based on a single measurement, not on many repeated measurements.

The bootstrap method for statistical uncertainty estimation is based on the *Re-sampling concept* – stating that the original sample represents the population from which it was drawn. Therefore, re-samples from the original sample should be equivalent to many samples taken from the original population. Bootstrapping a desired statistical variable is done by building its bootstrap distribution – drawing many of re-samples, with replacements, from the original sample, and evaluating the estimators for each re-sample. This will create the so-called bootstrap distribution. The standard deviation of the bootstrap distribution is then an estimator for the statistical uncertainty of the estimated value.

Bootstrap distributions include two sources of random variation:

1. Drawing the original sample at random from the original population.
2. Drawing re-samples at random from the original sample.

For an original sample that is large enough and re-sampled many times (hundreds or more), the random re-sampling adds very little variation compared to the variation due to the random choice of the original sample from the original population [5,4].

One can apply two basic criteria for checking if the re-sampling process is reliable and properly mimics the shape and spread of the original population. The first criterion – the bootstrap distribution approaches Gaussian distribution as the number of re-samples increases. The second criterion – the bootstrap distribution is unbiased, that is the mean of the bootstrap distribution minus the statistic of the original data is small.

3. Implementing bootstrap analysis for neutron multiplicity counting

To implement the bootstrap method on neutron multiplicity counting, we must first ask how do we define the original population? Clearly, the original sample must be the measurement taken. Thus, a good definition for the original population would be an infinitely long measurement. Clearly, obtaining an infinitely long measurement is not possible. But this dose not pose any real issues for two reasons: first, whenever statistical estimations are done, this means that the full distribution is not viable. Second, once the statistical error is sufficiently small (say, smaller than our measurement capabilities), the measurement may be considered "infinitely long". Thus, the term *infinitely long* may be replaced by *sufficiently long*.

But even when both the original sample and the original population are defined, it is still not obvious how the re-sampling process should be done. When measuring, for instance, the average height of a population, both the original population and the original sample are constructed of *individuals*, and the re-sampling is done by creating permutation of the individuals in the sampled population.

The bootstrapping procedure suggested in [2], which is also adopted in the present study, is as follows: Assume that the measurement was taken in the time interval $\mathcal{I} = [0, T]$, and let

$$T_l = \frac{l}{N}T, \quad l = 0 \ldots N \tag{1}$$

denote $N+1$ intermediate times points. To implement bootstrap analysis, we divide the original measurement to N sub-measurements, where the lth sub-measurement is defined as the original measurement between times $\mathcal{I}_\ell = [T_{\ell-1}, T_\ell)$, $\ell = 1 \ldots N$. The duration $\frac{T}{N}$ will be referred to as the *bootstrap gate* (The bootstrap procedure presented in [1] is not exactly the same as the present one; In [1], re-sampling was done directly on the counts and not on the original data. In particular, the bootstrap gate was equal to the multiplicity gate, since access to LIST-mode data was not