Composites Part B 79 (2015) 535–552

Contents lists available at ScienceDirect

Composites Part B

journal homepage: www.elsevier.com/locate/compositesb

# Nonlocal and shear effects on column buckling of single-layered membranes from stocky single-walled carbon nanotubes

Keivan Kiani*

*Department of Civil Engineering, K.N. Toosi University of Technology, Valiasr Ave., P.O. Box 15875-4416, Tehran, Iran*

ARTICLE INFO

*Article history:*
Received 27 February 2015
Received in revised form
17 April 2015
Accepted 20 April 2015
Available online 29 April 2015

*Keywords:*
A. Nano-structures
B. Buckling
B. Elasticity
C. Analytical modeling
Membranes of single-walled carbon nanotubes

ABSTRACT

Axial buckling behavior of single-layered membranes from vertically aligned single-walled carbon nanotubes is studied in the context of the nonlocal continuum theory of Eringen. To this end, useful discrete models based on the nonlocal Rayleigh, Timoshenko, and higher-order beam theories are developed to evaluate critical buckling loads associated with both in-plane and out-of-plane buckling modes. In discrete models, the size of the eigenvalue equations to be solved drastically magnifies for highly populated membranes. Thereby, development of models whose computational efforts do not affected by the population of the membrane is of great advantageous. To bridge this scientific gap, appropriate nonlocal continuous models are established based on the developed discrete models. The accuracy of the proposed discrete and continuous models is checked and remarkable results are achieved. Subsequently, the roles of the influential factors on both in-plane and out-of-plane axial buckling loads are addressed. The obtained results can be regarded as a basic step in examining of axial buckling mechanisms of more complex systems consist of multi-layered membranes from parallel or even orthogonal single-walled carbon nanotubes.



## 1. Introduction

Since the discovery of carbon nanotubes (CNTs), they have been increasingly paid attention to by scientific communities due to their exceptional physical and mechanical properties. Single-walled carbon nanotubes (SWCNTs) are tubes of graphene with a single cylindrical wall. In the process of synthesis, membranes, forests or jungles of CNTs can be produced. In each of these nanosystems, each nanotube tightly interacts with its neighboring tubes due to existing van der Waals (vdW) forces. The vdW force is the main agent of the self-assemblage of physical structures comprising individual CNTs. Essentially, the stability and vibration of such a group of CNTs depends on the aspect ratio of individual CNTs, radius of CNTs, chirality, intertube distance, and configuration of individual CNTs with respect to each other. The ensembles of CNTs are expected to be building blocks of the upcoming nano-/micro-electromechanical systems (NEMS/MEMS) [1–4]. On the hand, stability control of such systems is a vital stage in the mechanical design process. To ensure regarding the safe transfer of the exerted loads on these nanosystems, the buckling of its constituents, particularly ensembles of CNTs, should be realized in advance. To achieve this goal, axial load-bearing capacity of a membrane of vertically aligned SWCNTs is aimed to be examined. The vertically aligned membranes of our interest are tightly closed parallel-in-plane tubes.

In order to capture accurate buckling behavior of CNTs, molecular mechanics approach is a good choice. For example, atomistic-based models have been implemented to examine axial buckling [5–9], torsional buckling [10–12], bending buckling [13,14], and their postbuckling analyses [15,16]. However, such methods would take a lot of labor and time. To overcome such drawbacks, appropriate continuum models have been developed. Using continuum-based models, load bearing capacity of CNTs has been investigated from various points including axial buckling [17–22], bending buckling [23,24], thermoelastic buckling [25–30], torsional buckling [10,31–35], and combined torsion and axial load [36]. The postbuckling behavior of CNTs has attracted the attention of nanotechnology scientists during the past decade [37,38]. Several of these works [17,18,29,32,33,36,38]

* Tel.: +98 21 88779473; fax: +98 21 88779476.
*E-mail addresses:* k_kiani@kntu.ac.ir, keivankiani@yahoo.com.

http://dx.doi.org/10.1016/j.compositesb.2015.04.030
1359-8368/

are based on the classical theory of elasticity which cannot capture the inter-atomic bonds. In order to include such an important issue in the formulations of the problem, several advanced continuum theories have been developed in the last century. These are also called size-dependent models. The nonlocal continuum field theory of Eringen [39–43] is one of the most successful theories that have been broadly applied in vibration behavior [44–58] and buckling analyses [19–21,27,28,30,31] of nanobeams, nanoplates, CNTs, and their ensembles. In this theory, the size-dependency is incorporated into the equations of motion via a so-called small-scale parameter. Commonly, this factor is determined by justification of the predicted dispersion curves by the nonlocal model and those of an atomic approach. The magnitude of the small-scale parameter depends on the chirality, end conditions, and aspect ratio of the CNTs [59]. Additionally, elastic modulus of two-dimensional single crystal bodies [60] and uniaxial buckling of beams with nanocoatings [61] have been examined using other continuum-based models accounting for the inter-atomic forces.

The previous studies were restricted to buckling analysis of just an individual SWCNT or multi-walled CNTs where the CNTs are concentric. Buckling of doubly parallel nanotubes has been recently investigated by Murmu and adhikari [62] using nonlocal Euler-Bernoulli beam theory. To study the problem for a more general configuration, Kiani [63] investigated axial buckling of an ensemble consists of vertically aligned SWCNTs in two orthogonal directions. In the performed study, nonlocal Euler-Bernoulli beam model was exploited, and in the case of membrane of SWCNTs, only in-plane interactional van der Waals (vdW) forces were considered. However, a recent study [56] showed that for more accurate prediction of vibrations of SWCNTs, the effects of both in-plane and out-of-plane vdW forces should be taken into account. Such a new insight to the physical nature of the problem is the basic motivation of the present work. Herein, axial buckling behavior of single-layered two-dimensional (2D) membranes made of SWCNTs is going to be investigated. Using nonlocal shear deformable beam models, useful discrete and continuous models are developed to examine both in-plane and out-of-plane buckling behaviors of the nanosystem. The explicit expressions of the axial buckling loads are derived. The efficiency of the proposed continuous models is displayed and the roles of influential factors on both in-plane and out-of-plane buckling loads are explained in some detail. The obtained results in the present work would be very helpful in the design and mechanical analysis of the upcoming NEMS based on the multi-layered membranes of SWCNTs.

$$U^R = \frac{1}{2}\sum_{i=1}^{N_z}\int_0^{l_b}\left(\begin{array}{c} -\frac{\mathrm{d}^2V_i^R}{\mathrm{d}x^2}\left(M_{bz_i}^{nl}\right)^R - \frac{\mathrm{d}^2W_i^R}{\mathrm{d}x^2}\left(M_{by_i}^{nl}\right)^R + N\frac{\mathrm{d}^2V_i^R}{\mathrm{d}x^2} + N\frac{\mathrm{d}^2W_i^R}{\mathrm{d}x^2} + \\ C_{vz}\left(\left(W_i^R - W_{i-1}^R\right)^2(1-\delta_{i1}) + \left(W_i^R - W_{i+1}^R\right)^2(1-\delta_{iN_z})\right) + \\ C_{vy}\left(\left(V_i^R - V_{i-1}^R\right)^2(1-\delta_{i1}) + \left(V_i^R - V_{i+1}^R\right)^2(1-\delta_{iN_z})\right) \end{array}\right)\mathrm{d}x, \quad (1)$$

## 2. Assumptions and description of the problem

Consider a membrane of $N_z$ vertically aligned SWCNTs in the vicinity of each other as shown in Fig. 1(a). The intertube distance is $d$, the length of SWCNTs is equal to $l_b$, and each nanotube interacts with its neighboring tubes by the in-plane and out-of-plane vdW forces. The transverse components of the interactional vdW forces between two adjacent tubes due to their transverse vibrations, for instance tubes 1 and 2, are calculated by: $\Delta F_y = C_{vy}\Delta V$ and $\Delta F_z = C_{vz}\Delta W$ where $\Delta V = V_1 - V_2$ and $\Delta W = W_1 - W_2$ in order denote the discrepancies between the in-plane and out-of-plane displacements of the considered tubes, $C_{vy}$ and $C_{vz}$ represent their corresponding coefficients of the vdW force [56]. As it is seen in Fig. 1(b), the roles of these constants in vibration of the nanosystem are exactly identical to in-plane and out-of-plane continuous springs whose constants are $C_{vz}$ and $C_{vy}$, respectively. The main assumptions in deriving these constants are: (i) the exerted vdW forces are uniformly exerted on each tube, (ii) only small transverse displacements are of concern. For a large deformation regime (for example when postbukling behavior of the nanosystem is of interest), the effects of higher-order displacements may become important and generally, cannot be ignored.

In continuum-based modeling of SWCNTs, each tube is modeled by an isotropic hollow circular cylindrical solid whose length and mean radius are identical to the length and radius of the parent SWCNT. Commonly, this is called equivalent continuum structure (ECS) and its wall's thickness is $t_b = 0.34$ nm. In fact, the geometry data of the ECS is chosen such that leads to the best correspondence between the dominant longitudinal, torsional, and flexural frequencies of the ECS and those of the parent SWCNT [64]. Since transverse vibrations of the membranes of vertically aligned SWCNTs are of particular interest, the equations of motion are established based on the appropriate beam models. In the present study, nonlocal Rayleigh beam theory (NRBT), nonlocal Timoshenko beam theory (NTBT), and nonlocal higher-order beam theory (NHOBT) are implemented for both discrete and continuous modeling of the nanosystem.

## 3. Buckling analysis of membranes from SWCNTs via discrete models

### 3.1. A discrete model on the basis of the NRBT

#### 3.1.1. Nonlocal governing equations

Using Rayleigh beam model in the framework of the nonlocal continuum theory of Eringen, the elastic strain energy of the considered membrane, $U^R$, is given by:

where $\delta_{ij}$, $(M_{by_i}^{nl})^R$, and $(M_{bz_i}^{nl})^R$ in order denote the Kronecker delta and the nonlocal bending moments of the $i$th nanotube about the $y$ and $z$ axes based on the NRBT. Using nonlocal continuum theory of