Composites Part B 79 (2015) 553–580

Contents lists available at ScienceDirect

Composites Part B

journal homepage: www.elsevier.com/locate/compositesb

A size-dependent third-order shear deformable plate model incorporating strain gradient effects for mechanical analysis of functionally graded circular/annular microplates

Bo Zhang [a, b, c], Yuming He [b, c, *], Dabiao Liu [b, c], Jian Lei [b, c], Lei Shen [b, c], Lin Wang [b, c]

[a] College of Hydraulic and Environmental Engineering, China Three Gorges University, Yichang 443002, China
[b] Department of Mechanics, Huazhong University of Science and Technology, Wuhan 430074, China
[c] Hubei Key Laboratory of Engineering Structural Analysis and Safety Assessment, Wuhan 430074, China

ARTICLE INFO

Article history:
Received 14 June 2014
Received in revised form
8 May 2015
Accepted 10 May 2015
Available online 19 May 2015

Keywords:
A. Plates
B. Buckling
C. Analytical modeling
C. Micro-mechanics

ABSTRACT

In this paper, we develop a novel size-dependent plate model for the axisymmetric bending, buckling and free vibration analysis of functionally graded circular/annular microplates based on the strain gradient elasticity theory. The displacement field is chosen by using a refined third-order shear deformation theory which assumes that the in-plane and transverse displacements are partitioned into bending and shear components and satisfies the zero traction boundary conditions on the top and bottom surfaces of the microplate. Besides, the present model contains three material length scale parameters to capture the size effect. The material properties of the microplate are assumed to vary in the thickness direction and estimated through the classical rule of mixture. By using Hamilton's principle, the equations of motion and boundary conditions are obtained. Afterward, the differential quadrature method is adopted to discretise the governing differential equations along with various types of edge supports and therefore the deflection, critical buckling load and natural frequency can be determined. Convergence and comparison studies are carried out to establish the reliability and accuracy of the numerical results. Finally, a parametric study is conducted to investigate the influences of material length scale parameters, gradient index, thickness-to-outer radius ratio, outer-to-inner radius ratio and boundary conditions on the mechanical characteristics of the microplate.

1. Introduction

Functionally graded materials (FGMs) are the advanced materials in the family of engineering composites that have continuous variation of material properties from one surface to another and thus eliminate the stress concentration existing in laminated composites. The concept of FGMs was first proposed in 1984 by a group of material scientists in Japan [1,2]. Typically, these materials are made from a mixture of ceramic and metal in which the ceramic component provides high-temperature resistance due to its low thermal conductivity and the ductile metal component prevents fracture caused by thermal stresses. In recent years, the application of FGMs has been spread broadly in micro- and nano-scale devices and systems such as thin films [3,4], atomic force microscopes (AFMS) [5], micro- and nano-electro- mechanical systems (MEMS and NEMS) [6]. Microbars, microbeams and microplates are the key components widely used in MEMS, NEMS and AFM with the order of microns or submicrons. In such applications, the size-dependent deformation behavior has been experimentally observed [7–11]. For example, Lam et al. [8] found that the bending rigidity increases about 2.4 times as the beam thickness reduced from 115 μm to 20 μm in the micro-bending testing of epoxy polymeric beams. By using a novel automated torsion balance, Liu et al. [9] conducted torsion tests on polycrystalline copper wires with diameter ranging from 18 μm to 105 μm and observed that the normalized torque corresponding to 18 μm is nearly 1.5 times as that of 105 μm.

Due to lacking additional material length scale parameters, classical continuum theory cannot interpret and predict the size effect on the mechanical behavior of micro-structures. For this reason, utilization of size-dependent continuum theories, such as the strain gradient elasticity theory [8], modified couple stress

* Corresponding author. Department of Mechanics, Huazhong University of Science and Technology, Wuhan 430074, China. Tel.: +86 27 87543438; fax: +86 27 87544882.
E-mail address: ymhe01@sina.com (Y. He).

http://dx.doi.org/10.1016/j.compositesb.2015.05.017
1359-8368/

theory [12], nonlocal theory [13], surface elasticity theory [14], and so on, is necessary. Among these theories, the modified couple stress theory proposed by Yang et al. [12] and strain gradient elasticity theory developed by Lam et al. [8] are the two most widely employed in modeling and analysis of the mechanical behavior of micro-structures. In the former theory, the strain energy density function depends on the strain tensor and the symmetric part of the rotation gradient tensor only. Therefore, only one material length scale parameter is included in the non-classical constitutive relation. Based on this theory, several size-dependent linear/nonlinear beam, plate and even shell models proposed for the static bending, buckling and vibration problems of microstructures [15–39] made of homogeneous materials or FGMs. For example, Park and Gao [15] and Kong et al. [16] studied the static and dynamic characteristics of Bernoulli-Euler microbeams and derived the corresponding analytical solutions for the simply supported microbeam and cantilever microbeam respectively. Ma et al. [17] presented a modified couple stress Timoshenko beam model which incorporated the Poisson's effect and solved the static bending and free vibration problems of a simply supported microbeam by using Navier method. Tsiatas [18] developed a size-dependent Kirchhoff plate model for the static bending analysis of microplates with arbitrary shape and solved the boundary value problem through the method of fundamental solutions. To address the free vibration problem, Jomehzadeh et al. [20] presented closed form solutions for rectangular microplates with two opposite edges simply supported and arbitrary boundary conditions along the other edges and circular microplates with arbitrary boundary condition at circular edge. Considering the transverse shear deformation effect, Ma et al. [21] and Ke et al. [23] developed a size-dependent shear deformable plate model to examine the bending and free vibration characteristics of moderately thick microplates respectively. Ghayesh and Farokhi [35] examined the nonlinear motion characteristics of an extensible third-order shear deformable microbeam with an intermediate spring-support for the first time. Eshraghi et al. [37] put forth modified couple stress theory based modeling and analysis techniques for functionally graded circular/annular microplates, which taken into account the spatial variation of the length scale parameter.

Compare to the modified couple stress theory, the strain gradient elasticity theory is more extensive and applicable to capture the size effect since it not only contains the rotation gradient tensor but also the dilatation and deviatoric stretch gradient tensors. Thus, three material length scale parameters and two classical Lame's constants are included in the constitutive relation. By setting two of the three material length scale parameters to zero, the modified couple stress theory can be recovered. In recent years, the application of strain gradient elasticity theory has attracted considerable attention in the investigation of the mechanical behavior of microbars, microbeams and microplates. For instance, Kong et al. [40] and Wang et al. [41] studied the bending and vibration characteristics of Bernoulli-Euler and Timoshenko microbeams. Subsequently, Kahrobaiyan et al. [42] and Zhang et al. [43] proposed strain gradient Bernoulli-Euler and Timoshenko beam elements respectively. Li et al. [44] developed a strain gradient bilayered Bernoulli-Euler beam model in which the neutral axis and zero-stress axis no longer coincided with each other and their locations exhibited obvious size effect. Ansari et al. [45,46] studied the linear/nonlinear vibration characteristics of FG Timoshenko microbeams. Akgöz and Civalek [47] investigated the longitudinal free vibration behavior of axially functionally graded microbars. Rayleigh-Ritz technique was utilized to obtain an approximate solution to the longitudinal free vibration problem of microbars with clamped–clamped and clamped-free boundary conditions. Wang et al. [48] developed a stain gradient Kirchhoff plate model and investigated the static bending, free vibration and buckling behaviors of a simply supported microplate. Li et al. [49] further established an analytical model for the elastic bending problem of a bi-layered Kirchhoff microplate. Considering the effect of the longitude magnetic field, Jamalpoor and Hosseini [50] conducted the biaxial buckling analysis of double-orthotropic Kirchhoff microplate-systems including in-plane magnetic field for three different cases and coupled by an internal elastic medium. Ansari et al. [51] presented a strain gradient Mindlin plate model to predict the axisymmetric bending, buckling, and free vibration characteristics of moderately thick FG circular/annular microplates. Although the first-order shear deformation theory gives sufficiently accurate result for moderately thick microplates, it is not convenient to use due to it requires a shear correction factor which is difficult to determine since it depends on many parameters. Thus, it is necessary to adopt higher-order shear deformation theories for the mechanical analysis of thick microplates. Recently, several size-dependent higher-order shear deformable beam and plate models which satisfy the zero traction boundary conditions on the top and bottom surfaces of the micro-structures have been established by Zhang and his co-authors [52–55], Akgöz and Civalek [56,57] and Sahmani and Ansari [58,59]. For example, Zhang et al. [54] proposed an improved third-order shear deformable beam model for static bending, buckling and free vibration analysis of FG microbeams embedded in elastic medium. Based on a refined deformation theory with containing four unknowns, Zhang [55] developed an efficient size-dependent plate theory for bending, buckling and free vibration analysis of functionally graded microplates resting on elastic foundation. Moreover, the preliminary works on the modeling and analysis of microshell structures have been conducted by some researchers. For instance, Zeighampour and Tadi Beni [60] established a strain gradient cylindrical thin shell model to study the free vibration behavior of a single-walled carbon nanotube with simply supported boundary conditions. Zhang et al. [61] presented a four-unknown shear deformable FG cylindrical shell model based on the strain gradient elasticity theory and derived closed-form solutions of the free vibration problem of thick cylindrical microshells with simply supported ends and fixed ends.

From the above literature review, it is clearly found that the current study mainly focuses on the modeling and analysis of straight microbeams and rectangular microplates. However, the investigations involved curved microbeams, circular/annular microplates and microshells are still very limited. The objective of this paper is to develop a size-dependent plate model for the axisymmetric bending, buckling and free vibration analysis of FG circular/annular microplates based on the strain gradient elasticity theory. The displacement field is chosen based on a refined third-order shear deformation theory [62] which assumes that the in-plane and transverse displacements consist of bending and shear components and satisfies the zero traction boundary conditions on the top and bottom surfaces of the plate. The rest of this paper is structured as follows. In Section 2, the mathematical idealization of FGMs is reviewed. In Section 3, the equations of motion and boundary conditions of FG circular/annular microplates are derived by using Hamilton's principle. In Section 4, the solution procedure of differential quadrature (DQ) is applied to determine the deflection, natural frequency and critical buckling load. In Section 5, the convergence, comparison and parameter studies are performed. The major conclusions are summarized in Section 6.

2. Homogenization of material properties

Fig. 1 depicts the schematic diagram of an annular microplate with inner radius R_a, outer radius R_b and uniform thickness h. Obviously, a circular microplate can be viewed as the special case of