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# Nano piezoelectric/piezomagnetic energy harvester with surface effect based on thickness shear mode

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ABSTRACT

*Background:* Energy harvesters with piezoelectric materials are widely discussed for the new kinds of smart structures. However, reports on the energy harvesters at the nano scale which have large potential applications in the future are rather limited.
*Methods:* It's well known that the surface or interface stress can affect the mechanical properties of nanostructures. This work proposes the nano energy harvester with piezoelectric/piezomagnetic structure, in which the thickness-shear mode is considered by the surface stress model.
*Results:* The vibration motion and output power density are derived and calculated. The peak value of the power density can be enlarged by increasing the residual surface stress and the surface effect on the nano-plate energy harvester can be influenced by both the surface piezoelectric and piezomagnetic elastic constants. Moreover, the harvesting ability can be improved by increasing the thickness of the piezoelectric layer.
*Conclusion:* The capability of the energy harvester depends on the residual surface stress and the surface material constants. The proposed model provides the possibility of applying nano composite structures to the energy harvester.



## 1. Introduction

Nowadays, the nanomaterials and nanostructures are extensively applied on manufacture, information technology, bioengineering, energy and environment, etc. [1–4]. It's well known that the surface or interface stress can affect the mechanical properties of nanostructures. Gurtin et al. [5,6] presented a continuum surface elastic model to study the surface or interface effect. Then a lot of attention has been focused on the mechanical characteristics of nanostructures with the surface stress model and more information and application about such theory can be found by Refs. [7–13].

On the other hand, with the electro-mechanical coupling of the enhanced piezoelectric effect, piezoelectric nanostructures have strong potential applications to sensors, resonators, generators and transistor in the nanoelectromechanical systems [14–16]. Similar to the traditional nanostructures, surface stress has obvious influences on mechanical behaviors of nano piezoelectric system. Then the static and dynamic characteristics of nano piezoelectric structures can be further developed and extensively presented by the surface effect theory [17–19].

Energy harvesters are smart systems to gain energy from working environment for microelectronic devices, which have been widely used in modern sensing technology. Recently, an increasing efforts have been paid on the piezoelectric energy harvesters with extensional, bending and thickness-shear modes [20–22]. Furthermore, with the coupling of the mechanical, electric and magnetic fields, a new kind of energy harvesters consisting of magnetoelectricelastic materials appear in quite recent years and show its superior properties [23,24]. However, researches on such energy harvesters are very limited. In this paper, the nano plate-form energy harvester with magnetoelectricelastic materials are reported and studied. By the surface stress theory, dynamical characteristics are presented and some new and interesting phenomena are presented.

## 2. Basic equations

Consider a rectangular nano plate-form energy harvester composed of the piezomagnetic and piezoelectric layers shown in Fig. 1. The thicknesses of the piezomagnetic and piezoelectric layers are $h_1$ and $h_2$. The lengths along the $x$- and $z$-direction are assumed to be much larger than that along the $y$-direction. The poling directions of the piezomagnetic and piezoelectric layers are both along the $z$-axis and two electrodes are installed in the two surfaces

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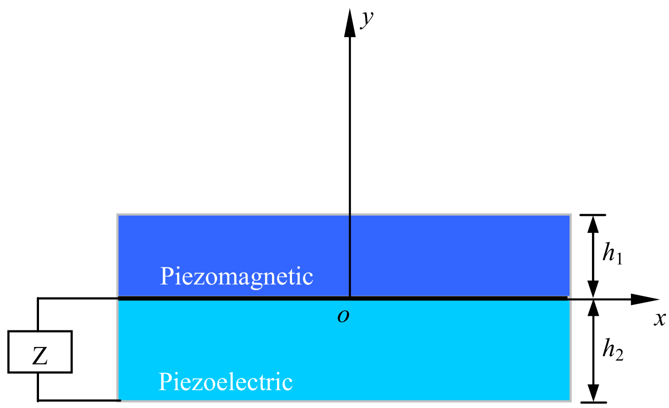


**Fig. 1.** Structural model for a nano plate-form energy harvester with surface effect.

of the piezoelectric layer. A load circuit with the impedance $Z$ is connected to the two electrodes. Moreover, the surfaces and the interface of the piezomagnetic and piezoelectric materials are considered to be the thin membranes so that the thicknesses are not considered.

The elastic displacement, electric potential and magnetic potential are

$$\varphi = \varphi(y,t),\ \psi = \psi(y,t),\ u_z^e = u_z^e(y,t),\ u_z^m = u_z^m(y,t), \tag{1}$$

where $\phi$ and $\psi$ are the electric potential and magnetic potential, $u_z^e$, $u_z^m$ are the elastic displacements along the $z$-axis for piezoelectric and piezomagnetic layers. The superscripts '$e$' and '$m$' are used to represent piezoelectric and piezomagnetic materials, respectively.

For the piezoelectric and piezomagnetic layers, the stress, electric displacement and magnetic flux are

$$T_{yz}^e = c_{44}^e \frac{\partial u_z^e}{\partial y} + e_{15}\frac{\partial \phi}{\partial y}, \quad D_y = e_{15}\frac{\partial u_z^e}{\partial y} - \varepsilon_{11}\frac{\partial \phi}{\partial y}, \tag{2a, b}$$

$$T_{yz}^m = c_{44}^m \frac{\partial u_z^m}{\partial y} + h_{15}\frac{\partial \psi}{\partial y}, \quad B_y = h_{15}\frac{\partial u_z^m}{\partial y} - \chi_{11}\frac{\partial \psi}{\partial y}, \tag{3a, b}$$

where $c_{44}^e$, $e_{15}$, $\varepsilon_{11}$ are the elastic, piezoelectric and dielectric constants for the piezoelectric materials and $c_{44}^m$, $h_{15}$,$\chi_{11}$ are the elastic, piezomagnetic constants and magnetic permeability for the piezomagnetic structure.

At the surface and interface, the constitutive relations can be expressed as

$$T_{yz}^{e(s)} = T_{yz}^0 + c_{44}^{e(s)}\frac{\partial u_z^e}{\partial y} + e_{15}^{(s)}\frac{\partial \phi}{\partial y}, \tag{4}$$

$$T_{yz}^{m(s)} = T_{yz}^0 + c_{44}^{m(s)}\frac{\partial u_z^m}{\partial y} + h_{15}^{(s)}\frac{\partial \psi}{\partial y}, \tag{5}$$

where $T_{yz}^{e(s)}$ and $T_{yz}^{m(s)}$ are the surface stresses of piezoelectric and piezomagnetic layers, $T_{yz}^0$ the residual surface stress, the superscript '$(s)$' stands for the surface layers.

The governing equation of the piezoelectric and piezomagnetic layers can be expressed as the following form:

$$c_{44}^e \frac{\partial^2 u_z^e}{\partial y^2} + e_{15}\frac{\partial^2 \phi}{\partial y^2} + \omega^2 \rho^e u_z^e = 0, \tag{6a}$$

$$e_{15}\frac{\partial^2 u_z^e}{\partial y^2} - \varepsilon_{11}\frac{\partial^2 \phi}{\partial y^2} = 0, \tag{6b}$$

$$c_{44}^m \frac{\partial^2 u_z^m}{\partial y^2} + h_{15}\frac{\partial^2 \psi}{\partial y^2} + \omega^2 \rho^m u_z^m = 0, \tag{7a}$$

$$h_{15}\frac{\partial^2 u_z^m}{\partial y^2} - \chi_{11}\frac{\partial^2 \psi}{\partial y^2} = 0. \tag{7b}$$

where $\omega$ is the circular frequency, $\rho^e$ and $\rho^m$ the mass densities of the piezoelectric and piezomagnetic materials.

Substituting Eq. (6b) into Eq. (6a), we can derive that

$$\frac{\partial^2 u_z^e}{\partial y^2} + (\lambda^e)^2 u_z^e = 0, \tag{8}$$

where

$$\lambda^e = \omega\sqrt{\frac{\rho^e \varepsilon_{11}}{c_{44}^e \varepsilon_{11} + e_{15}^2}}. \tag{9}$$

The solution of Eq. (8) is

$$u_z^e(y) = A_1 \cos(\lambda^e y) + A_2 \sin(\lambda^e y), \tag{10}$$

where $A_1$, $A_2$ are the constants to be determined.

Then, the solution of Eq. (6b) is

$$\phi = \frac{e_{15}}{\varepsilon_{11}}[A_1 \cos(\lambda^e y) + A_2 \sin(\lambda^e y)] + A_3 y + A_4, \tag{11}$$

where $A_3$ and $A_4$ are the undetermined constants.

Similarly, the elastic displacement of the piezomagnetic layer can be derived that

$$u_z^m(y) = A_5 \cos(\lambda^m y) + A_6 \sin(\lambda^m y), \tag{12a}$$

$$\psi = \frac{h_{15}}{\chi_{11}}[A_5 \cos(\lambda^m y) + A_6 \sin(\lambda^m y)] + A_7 y + A_8, \tag{12b}$$

where $A_5$, $A_6$, $A_7$ and $A_8$ are the constants to be determined and

$$\lambda^m = \omega\sqrt{\frac{\rho^m \chi_{11}}{c_{44}^m \chi_{11} + h_{15}^2}}. \tag{13}$$

Then we can derive that

$$T_{yz}^e = \lambda^e\left(c_{44}^e + \frac{e_{15}^2}{\varepsilon_{11}}\right)[A_2 \cos(\lambda^e y) - A_1 \sin(\lambda^e y)] + A_3 e_{15}, \tag{14a}$$

$$T_{yz}^{e(s)} = T_{yz}^0 + \lambda^e\left(c_{44}^{e(s)} + \frac{e_{15}^{(s)2}}{\varepsilon_{11}^{(s)}}\right)[A_2 \cos(\lambda^e y) - A_1 \sin(\lambda^e y)] + A_3 e_{15}^{(s)}, \tag{14b}$$

$$T_{yz}^m = \lambda^m\left(c_{44}^m + \frac{h_{15}^2}{\chi_{11}}\right)[A_6 \cos(\lambda^m y) - A_5 \sin(\lambda^m y)] + A_7 h_{15}, \tag{15a}$$

$$T_{yz}^{m(s)} = T_{yz}^0 + \lambda^m\left(c_{44}^{m(s)} + \frac{h_{15}^{(s)2}}{\chi_{11}^{(s)}}\right)[A_6 \cos(\lambda^m y) - A_5 \sin(\lambda^m y)] + A_7 h_{15}^{(s)}. \tag{15b}$$

The boundary conditions can be given as

$$y = -h_2: \quad T_{yz}^{e(s)}(-h_2) = 0, \tag{16a}$$