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A scintillation light radial distribution model for monolithic crystal gamma cameras

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A R T I C L E I N F O

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A B S T R A C T

Gamma imaging systems are often based on monolithic scintillators coupled to a light detector through a certain number of light guides, like glass or grease. In order to predict and optimize gamma camera performance in terms of bias and spatial resolution, accurate Monte Carlo simulations are usually carried out. These kind of simulations require high computational time so only a small number of arrangements are investigated. In this work we propose a mathematical model of scintillation light propagation starting from the gamma interaction point to the detector surface. This model gives the expression for the radial photon density distribution with a computational time 4 order of magnitude shorter than a Monte Carlo simulation. The model has been validated with standard Monte Carlo simulations of five system configurations, showing a percentage error on the quantitative evaluation of photon number lower than 2%.

1. Introduction

In gamma rays imaging applications (like SPECT and PET) the typical detector is a scintillation crystal coupled, through a light guide, to a photodetector system [1–3]. In order to estimate the interaction position of each single gamma ray within the crystal, the photodetector system must sample the scintillation photon distribution in different points of the detection area. Performance and cost of a gamma camera mainly depend on the features of the photodetector and on the combination of characteristics of the other components such as type and width of the crystal, number, type and width of the light guides, etc. [4]. Accurate and reliable simulations of such optical systems are typically performed by Monte Carlo simulations [5–8]. However, the computational requirements are very high and calculations can be very time consuming and the investigation of large number of types of optical systems is often unpractical. Therefore different approaches are required for the design and engineering of a gamma camera.

Starting from a previous work [9], in this paper we report on a mathematical model, based on geometrical optics laws. Our model describes a radial scintillation light distribution on the detection surface in the case of a continuous scintillation crystal coupled through an arbitrary number of light guides to the detector. Such model performs a fast evaluation of the Point Spread Function (PSF) distribution for different depths of interaction (DOI) of the gamma photons, different materials and geometrical characteristics of the crystal and of the light guides, allowing a faster evaluation of the corresponding performance. The proposed model provides the light distribution on the detector surface in terms of optical photon density. Therefore we can predict the number of detected optical photons. Simpler models of light distributions were found by [10,11] but they do not take into account the light reflection and the refraction effects with the interfaces.

After the photon density evaluation on the detector surface, the effect of finite statistics of scintillation photons from the modelled distribution has been considered, obtaining their final random coordinates in agreement with those of a Monte Carlo simulation, with a computational time, for the proposed cases, four order of magnitude shorter than a Monte Carlo simulation with the same number of scintillation events. For a more complex system, or for a greater number of events, the difference in terms of computational time between a Monte Carlo simulation and the proposed model will be greater. In order to validate the model, the computed light distributions have been compared with light distributions obtained from accurate GEANT4 Monte Carlo simulations [12–14] for five system configurations. The results demonstrate a good agreement. Finally, the use of estimation methods of scintillation position applied to model data, rather than simulated data [15,16], leads to a low

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computational time performance evaluation of the crystal/light guides/photo-detector system in terms of linearity and intrinsic spatial resolution.

2. Model of light propagation in the crystal

In order to study mathematically the propagation of scintillation light in a gamma camera several assumptions were made:

- Isotropic scintillation at an exact point of the crystal (at a given DOI, therefore) is assumed.
- Top and bottom surfaces of the crystal are considered perfectly polished in order to take into account only Snell–Descartes reflections and refractions [17].
- Absence of interference of the scintillation light is assumed.
- Monochromatic scintillation light is assumed (absence of chromatic dispersion phenomena).
- Edge walls of crystal and of optical guides are considered totally absorbing or very far from the scintillation point.

The last assumption has been made in order to find a simple radial model of light distribution on the detection surface. In fact if we considered reflections on the edge walls of the system, the circular symmetry of the light distribution would be broken. In Section 5, we will explain how to take into account reflection effects on the edge walls by using the found radial model. All the assumptions apply throughout this work. The reflectance (and transmittance) to the crystal glass (on the bottom face) is considered independent of the angle of incidence of light initially in this section, so that we can find a simple closed-form expression for the light intensity as showed in Eq. (3). For the evaluation of the radial light profile on the surface of the detector, the dependence of the reflectance (and transmittance) from the angle of incidence of light will be considered (Sections 3 and 4).

According to Lambert law (Eq. (1)), the intensity of the light produced by a point source on a surface is proportional to the cosine of the angle of incidence of light ray with respect to the normal to the surface (θ), and is inversely proportional to the square of the distance from the source (r):

$$I(\theta) = P_{st}\frac{\cos\theta}{r^2}e^{-r/\lambda_1} \quad (1)$$

In Eq. (1) the coefficient P_{st} represents the power radiated per unit solid angle. The exponential factor has been added to consider the scintillation light self-absorption in the crystal (λ_1 is the path length in the crystal). This formula can be rewritten as a function of the distance of the point source from the bottom of the crystal (h) and of the radial coordinate on the analyzed surface (x). $I_c(x)$ represents the light intensity radial profile on the bottom of the crystal

$$I_c(x) = \frac{P_{st}\cdot e^{-(\sqrt{x^2+h^2})/\lambda_1}}{h^2\left[1+\left(\frac{x}{h}\right)^2\right]^{3/2}} \quad (2)$$

Considering the upper interface of the crystal (see Fig. 1), the formula of the intensity must take account of all possible multiple reflections of light that contribute in the point x. In this case, the expression becomes the following:

$$I_c(x) = P_{st}\sum_{n=0}^{\infty}\frac{(\mathcal{R}_{bottom}\mathcal{R}_{top})^n\cdot e^{-(\sqrt{x^2+(2nl+h)^2})/\lambda_1}}{(2nl+h)^2\left[1+\left(\frac{x}{2nl+h}\right)^2\right]^{3/2}}$$

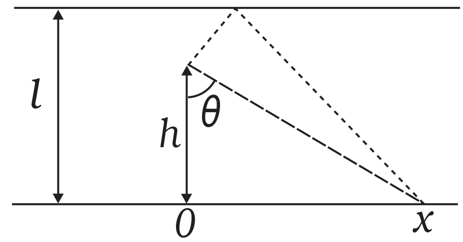

Fig. 1. Most significant contributions of the light (direct light and once reflected light) within the scintillator that reach the bottom face of the crystal to be forwarded to the next material.

$$+\frac{P_{st}}{\mathcal{R}_{bottom}}\sum_{n=1}^{\infty}\frac{(\mathcal{R}_{bottom}\mathcal{R}_{top})^n\cdot e^{-(\sqrt{x^2+(2nl-h)^2})/\lambda_1}}{(2nl-h)^2\left[1+\left(\frac{x}{2nl-h}\right)^2\right]^{3/2}}. \quad (3)$$

In Eq. (3) each contribution of the summations is similar to Eq. (2) with h substituted for $(2nl+h)$ or $(2nl-h)$, as the specular Snell reflection can be modelled by placing a virtual source farther from the bottom surface of the crystal than the real source without considering the presence of the top surface of the crystal. The virtual source distance from the bottom surface of the crystal is chosen to assure the equality of the optical lengths travelled by the real source reflected light and the virtual source light. The first summation refers to contributions due to an even number of reflections with the upper and lower interface (the contribution of the direct light is considered with $n=0$), while the second takes into account contributions in which the light is reflected an odd number of times. We define the bottom surface of the crystal as the closer crystal face to the detector, while the top of the crystal is the farther face from the detector. $\mathcal{R}_{bottom}$ and $\mathcal{R}_{top}$ respectively represent the reflectance on the top and bottom surface of the crystal. l is the thickness of the crystal and n is the index of summation that distinguishes the various contributions of the reflected light.

Eq. (3) can be approximated by the direct light contribution and by the first N light reflection contributions. Normally it is well approximated by the first terms of each sum. Therefore we get

$$I_c(x) = \frac{P_{st}\cdot e^{-(\sqrt{x^2+h^2})/\lambda_1}}{h^2\left[1+\left(\frac{x}{h}\right)^2\right]^{3/2}} + \frac{P_{st}\cdot\mathcal{R}_{top}\cdot e^{-(\sqrt{x^2+(2l-h)^2})/\lambda_1}}{(2l-h)^2\left[1+\left(\frac{x}{2l-h}\right)^2\right]^{3/2}}. \quad (4)$$

The scintillation process happens in a short period of time, therefore we can express all the light intensities (I) in terms of photon number densities ($\mathcal{N}$), substituting P_{st} for N_{st}. N_{st} is the average number of photons radiated per unit solid angle during the scintillation process. It can be expressed as

$$N_{st} = \frac{E_g\cdot LY}{4\pi}, \quad (5)$$

where E_g is the gamma ray energy and LY is the light yield of the scintillator.

3. Model of light propagation with one optical guide

In Section 2 the functional dependence of light radial distribution on the bottom of the crystal, before it is transmitted over the crystal-light guide interface, was found. To obtain the radial light distribution on the detection surface placed at the end of a single light guide, a numerical analysis was performed. It is based on a more accurate model than the one adopted in the previous section. The transition to