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An analytical solution to the problem of linear track segment reconstruction in drift tube chambers

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ABSTRACT

Drift tube chambers are used for high resolution position measurements of charged particles in high energy physics. In the case of track reconstruction in two dimensions, for given geometry of a calibrated drift tube chamber, its output can be translated to a collection of triples, each one containing the coordinates of the wire of the drift tube crossed by a particle and the distance of the closest approach of a particle to the wire. The distance of the closest approach defines a drift circle which has the center at the wire and the radius equal to the distance of the closest approach. To reconstruct a linear segment of a track, a common tangent must be found to drift circles associated with that single track. In the study it is shown that this problem can be solved analytically.

1. Introduction

Particle detectors are indispensable in any experiment in high energy physics. Modern detectors are usually designed as sampling detectors—the volume of a detector is filled with devices recording events related to particles crossing the device. The actual physical phenomena which underlay the functioning of such a device can be for example an excitation in a solid-state detector, a primary ionization in a gaseous chamber or an energy deposition in a sensitive volume of a calorimeter.

Among other types of detectors are drift tube chambers used for high resolution position measurements of charged particles. The drift tube chambers are build of individual drift tubes filled with a gas mixture. The actual composition of the mixture influences the properties of the detector. An individual drift tube is a metallic cathode cylinder and an anode wire at the center of the cylinder. A charged particle crossing the tube ionizes the gas along its path. One may distinguish between primary ionization caused by the fast particle crossing the drift tube and secondary ionization caused by the ionization electrons. The ionization electrons drift towards the wire, where they are amplified in avalanches and collected. Then the electronics of the detector records the pulse from the anode and assigns to this event the coordinates of the centre of the drift tube. Additional quantities can be also measured, among them the drift time which is the time interval between the anode wire pulse and some trigger pulse.

The length of the drift time can be used to determine the distance of the closest approach of a particle to the anode wire. For this purpose, a calibration formula must be found which relates the drift time of the electrons to the distance between the particle's track and the anode wire. Numerous effects lead to a non-linear drift-time–drift distance relation, and the spatial resolution depends on the distance of the closest approach of the particle to the wire. Given the distance of the closest approach, each drift tube signal can be depicted as a drift circle with the center at the anode wire. For a single track passing through the chamber a particle's track is just the common tangent to a collection of drift circles recorded in all layers of the drift chamber detector (Fig. 1). Comprehensive description of the topic of particle detection in drift chambers can be found in Ref. [1].

The problem of finding the common tangent to a set of circles is of vital importance for accurate reconstruction of particles in experiments of high energy physics. Besides being accurate, an algorithm for track fitting should be also fast. Global methods for track fitting use all collected data to find track parameters. Among these methods are the ones based on Hough transform [2] and Legendre transform [3]. Both methods use the collected data to construct histograms in some parameter space. Then the maxima of the histograms define the trajectories in the physical space. In contrast to the Hough transform, the drift time measurement is directly used in the Legendre transform-based method for determining the tangent common to a set of drift circles. Because histogramming is performed in a digitized parameter space, none of the aforementioned methods achieves the accuracy of an approach based on the straightforward least squares formulation of the problem (e.g. [4–7]). According to this formulation (described in detail in the next section) in an ideal case the

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distance of the closest approach of a particle to the anode wire should be equal to the radius of the drift circle. Consequently, the problem of determining the unknown parameters of the linear segment of a particle's trajectory can be reduced to a problem of finding minimum of some objective function. Yet another approach to the problem of common tangent fitting is based on the method of Kalman filter [8]. Frequently, Kalman filtering is an intermediate step within some combinatorial framework designed to find acceptable continuations of already found track parts [9].

All aforementioned methods require substantial numerical work. In the present paper it is shown that the problem of fitting the linear segment to drift tube data can be solved analytically. Closed form formulas are given for the solution. An appropriate heuristic is proposed to translate the least squares problem to a problem which can be tackled analytically.

2. Description of the method

In the case of drift tubes the input data for the particle tracking module has a form of triples $\{X_i,Z_i,R_i\}:i=1\ldots N$, where (X_i,Z_i) is the center of the ith tube (an anode wire) crossed by a particle, and R_i is the distance of the closest approach of the particle to the wire of the ith drift tube (R_i is determined from the measurement of the drift time t, based on the R–t calibration curve). For a linear segment of a particle track the task is to find a straight line $L: x=Az+B$ which for each i passes at distance R_i from (X_i,Z_i). The distance d_i of (X_i,Z_i) from L is equal to:

$$d_i=\frac{|AZ_i+B-X_i|}{\sqrt{1+A^2}} \tag{1}$$

The problem of determining the slope A and intercept B of L can be formulated as the problem of minimizing the sum of squares $S(A,B)$ of the difference between R_i and d_i:

$$S(A,B)=\sum_{i=1}^{N}\left(\frac{|AZ_i+B-X_i|}{\sqrt{1+A^2}}-R_i\right)^2. \tag{2}$$

Obviously, A and B can be found from the requirement that the partial derivatives of $S(A,B)$ with respect to A and B are equal to zero. While numerical solution of this problem is straightforward, it appears that finding the analytical solution is a harder problem because derivatives of $S(A,B)$ cannot be computed analytically if the signs of the differences AZ_i+B-X_i are not known, and these signs are not known because A and B are not known.

In fact it means that an appropriate heuristics is necessary which will assign the centers of the drift tubes (X_i,Z_i) to either right or left sides of L. Below such a heuristic method is proposed and it is shown how it can be used for the analytical solution of the least squares problem of interest. The quality of the heuristics-based analytical solution is compared to the numerical solution in the next section.

To effectively solve the problem of finding analytically the slope A and the intercept B, an appropriate method to determine the signs of AZ_i+B-X_i in Eq. (2) must be provided. The proposed heuristics finds the heuristic straight line H which fits best to the measurement points $\{X_i,Z_i,R_i\}:i=1\ldots N$, where the drift radii R_i are identified with the measurement errors. Thus, the problem is reduced to finding the minimum of a heuristic function $S_H(A_H,B_H)$ with the respect to the slope A_H and the intercept B_H of the heuristic straight line $H: x=A_Hz+B_H$:

$$S_H(A_H,B_H)=\sum_{i=1}^{N}\left(\frac{A_HZ_i+B_H-X_i}{R_i}\right)^2. \tag{3}$$

The problem of finding A_H and B_H can be further reduced to a system of two linear equations:

$$\begin{pmatrix}\sum_{i=1}^{N}\frac{Z_i^2}{R_i^2} & \sum_{i=1}^{N}\frac{Z_i}{R_i^2}\\ \sum_{i=1}^{N}\frac{Z_i}{R_i^2} & \sum_{i=1}^{N}\frac{1}{R_i^2}\end{pmatrix}\begin{pmatrix}A_H\\ B_H\end{pmatrix}=\begin{pmatrix}\sum_{i=1}^{N}\frac{X_iZ_i}{R_i^2}\\ \sum_{i=1}^{N}\frac{X_i}{R_i^2}\end{pmatrix} \tag{4}$$

Given A_H and B_H one computes Δ_i^H as

$$\Delta_i^H=A_HZ_i+B_H-X_i. \tag{5}$$

Based on the sign of Δ_i^H the measurement points $\{X_i,Z_i,R_i\}$ will be assigned to either left or right side of the common tangent L.

The heuristic line H is used to split the set of measurement points $\{X_i,Z_i,R_i\}:i=1\ldots N$ into two separate classes: one on the left and one on the right side of L. In particular, assuming that the sign of Δ_i^H is the same as the sign of $\Delta_i=AZ_i+B-X_i$(the quality of this assumption is tested in the next section), the equation for $S(A,B)$ has the form:

$$S(A,B)=\sum_{AZ_m+B-X_m>0}\left(\frac{\Delta_m}{\sqrt{1+A^2}}-R_m\right)^2+\sum_{AZ_k+B-X_k<0}\left(\frac{\Delta_k}{\sqrt{1+A^2}}+R_k\right)^2, \tag{6}$$

If one sets $R_i<0$ for all $\Delta_i^H<0$ then the expression for $S(A,B)$ becomes

$$S(A,B)=\sum_{i=1}^{N}\left(\frac{AZ_i+B-X_i}{\sqrt{1+A^2}}-R_i\right)^2. \tag{7}$$

Given Eq. (7) we can compute B from the condition $\partial S(A,B)/\partial B=0$:

$$B=\frac{\sum_{i=1}^{N}X_i+\sqrt{1+A^2}\sum_{i=1}^{N}R_i-A\sum_{i=1}^{N}Z_i}{N}. \tag{8}$$

From $\partial S(A,B)/\partial A=0$ we get

$$\sum_{i=1}^{N}\left(\Delta_i-R_i\sqrt{1+A^2}\right)(Z_i+AX_i-AB)=0. \tag{9}$$

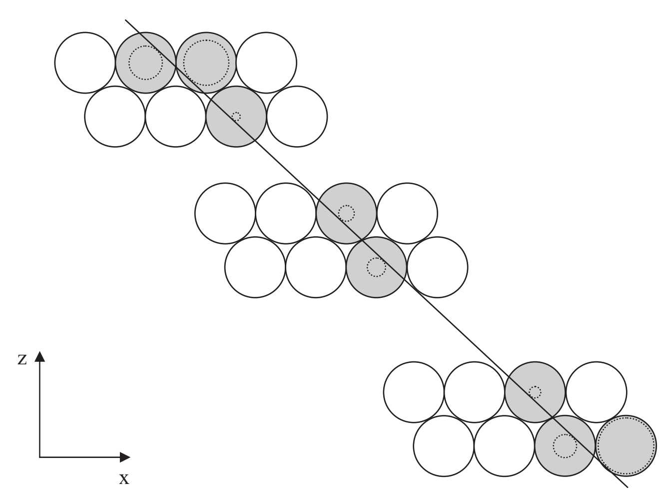

Fig. 1. The problem of fitting a common tangent to a set of circles. Solid circles are the drift tubes, circles filled with gray color are drift tubes crossed by linear segment of a particle's track. Dashed circles have the radius equal to the distance of the closest approach of a particle to a drift tube anode—their radii are calculated from R–t calibration curve.