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Analytical approach *à la* Newmark for curved laminated glass

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ABSTRACT

A method of solution that extends to the case of curved laminated structures the traditional approach developed by Newmark et al. for straight beams is presented. The method is specialized to curved laminated glass, a composite formed by two external glass layers that sandwich a very thin polymeric interlayer. The effect of curvature on the shear coupling of glass plies through the interlayer is examined in the paradigmatic example of a laminated beam with constant moderate curvature under radial loading with different boundary conditions, varying the initial camber, the end constraints and the elastic properties of the polymer. Comparisons with numerical experiments confirm the accuracy of the proposed modeling. In general the response of a curved structure is greatly influenced by the axial force it undergoes, and such internal action is mainly governed, for fixed applied loads, by the boundary conditions at the extremities. The axial force produces the arch-response of the structure, which is not substantially affected by the shear coupling of glass through the interlayer. On the other hand, such coupling has major effects on the bending properties.

1. Introduction

The type of sandwich structure considered here is formed by the composition of two thick and curved external layers, with membrane and bending stiffness, and a compliant core, much thinner than the external layers. The core does not present bending stiffness *per se*, but it can provide the shear coupling of the external layers. Moreover, it constrains the relative out-of-plane displacement of the external layers because, due the hypothesis of small thickness, one can consistently assume that its height does not change in the deformation. The modeling of composite laminated structures with a shear-compliant core is one of the most active research fields of the last decades [1–4]. The key role played by the interlaminar shear stress on the composite laminates response has been evidenced since the seminal works by Pagano [5–7] and Reddy [8,9]. Other works have focused on curved laminated beams composed by perfectly bonded plies with considerable bending stiffness [10–12]. The present article deals with the particular case of curved laminated glass, which represents a paradigmatic benchmark example for the use of theories of this kind.

Laminated glass is typically made of two glass plies bonded by a thermoplastic polymeric interlayer, with a treatment in autoclave at high pressure and temperature. Through lamination, safety in the post-glass-breakage phase is increased because fragments remain attached to the interlayer, reducing the risk of injuries. Its traditional use as an infill panel is the most popular, but in recent years its noteworthy load bearing capacity has been fully exploited in structural applications [13,14]. In the pre-glass-breakage phase, the polymeric interlayers can provide shear stresses that constrain the relative sliding of the glass plies [15,16]. Of course, the degree of coupling of the glass layers depends upon the shear stiffness of the interlayer [17]. Thus, the flexural performance is intermediate between the two borderline cases [18,19] of *monolithic limit* (shear-rigid bonding of the glass plies) and *layered limit* (free-sliding glass plies). Since stress and strain are much lower in the monolithic than in the layered limit, to avoid redundant design a large number of theoretical studies have been performed [20–25], corroborated by a wealth of experimental activity [26,14,27,28]. Several practical methods to readily calculate the response of laminated glass structures, such as those by Refs. [29,19,30,31] for the case of beams, and by Refs. [32–35] for the case of plates, have been proposed. A particular attention has also been paid to the buckling of laminated glass [36–40] and to hybrid laminated glass unit [41,42].

The aforementioned studies consider *flat* glass only, although *curved* laminated glass is increasingly being used in modern architecture to construct free-form roofs and façades. Curved glass is traditionally produced through hot-forming processes. In *sag*

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bending, flat plates are placed on a negative curved mold and then heated up to a temperature at which glass softens and becomes plastic. The action of gravity produces the uniform contact with the support and, when glass is cooled down, it retains the shape of the mold. Another technique is *press bending*, where glass is formed by using a stainless steel mold face and head that presses the panel into the shape of the mold. After cooling and hardening, the glass sheets must present mating curvatures. Polymeric interlayers are then placed between the sheets and the whole package is processed in autoclave for lamination. With this technique one can produce both single- and double-curvature panels, but a limitation is represented by the need of a dedicated mold for every shape, making this process attractive only for large quantities of identical panels.

In recent years, numerically-controlled machines allow to manipulate glass above the transition temperature and calender it to form single-curvature surfaces, whose radius can be arbitrarily varied in a continuous manner. Recent advances in geometric tassellation allow for the discretization of *any* surface using single curvature panels only, thus permitting to approximate free-form double curvature glazing with single-curvature panels [43–45]. Discretization of double curvature by single curved elements very often leads to slight modifications of the initial double curved 3D-shape in order to be able to panelise, but this method gives substantial economical advantages with respect to the production of hot-formed double-curvature panels, which always need a counterform. Because of the importance of this type of elements, it is therefore important to define reliable models for the calculation of single-curvature laminated-glass panels, which however are still far from a definite setting.

In fact, the approach of the practice for curved laminated glass usually neglects the effect of the curvature and uses the same equations as if the panel was flat, but this procedure is not corroborated by any theoretical consideration. Very recently, an analytical model specific for curved laminated glass beams has been proposed in Ref. [46]. In this approach there are however a few simplifying assumptions for the description of the kinematics of deformation of the layered beam. Moreover, the authors consider only the case in which the beam is an arch of a circle (constant curvature) and only radial loads are present. In Ref. [47], a general theory has been developed for curved three-layered beams, which well adapts to all those cases in which the Gaussian curvature of the glass surface is zero and the directions of principal curvature do not vary from point to point. In this article reference is made to the case of a rectangular laminated glass panel that has been hot-formed into a cylindrical shape with the generatrix parallel to one of the sides. Under the hypothesis that the applied actions produce bending in the same direction of the pre-formed camber, and if one neglects the effects of the transversal strains due to the lateral contraction, the beam model of [47] can be conveniently adapted for the structural analysis [2].

In architectural applications, the radii of curvature that are considered for roofs and façades are of the order of several meters, whereas the thickness of the glass plies is of the order of 10 mm. Consequently, it is not a restriction to assume that the ratio between the beam thickness and the radius of curvature is a quantity infinitesimal of the first order, and neglect higher order terms in the relevant expressions. Moreover, in most applications, the laminated glass elements are uniformly bent in the form of cylinders with constant mean curvature. As shown in Ref. [47], the hypotheses of moderate and constant curvature allow drastic simplifications in the governing equations. It is thus possible to analytically evidence the effect due to the curvature on the shear coupling of the external glass plies through the interlayer, and make comparisons with the case of straight beams.

In this article, starting from the general theory developed in Ref. [47], the governing equations are specialized to the case of laminated-glass beam with moderate constant curvature. In particular, a simple method of solution, based upon an extension of the original method first presented by Newmark et al. [48] for straight composite beams, is proposed as a practical tool to determine, in closed form, the response of curved beams under both tangential and radial loading. The method is extremely accurate and can be used in all those cases in which the bending moment and the axial force in the beam is *a priori* known, i.e., when the arch is statically determinate. This allows to reduce the set of equilibrium equations into a unique differential equation for the radial displacement only. Various paradigmatic examples of laminated-glass arches with constant moderate curvature are solved analytically under various load and boundary conditions. Comparisons with numerical experiments demonstrate the accuracy of the proposed approach.

2. Beams with moderate constant curvature sandwiching a thin core

Consider the case indicated in Fig. 1. The basic assumptions of the proposed model are the followings.

- The materials are linearly elastic, homogeneous and isotropic;
- strains, displacements and rotations are small;
- all the plies are assumed to be prismatic (i.e., constant cross-section) and geometrically planar, i.e., a plane exists that contains the undeformed axis;
- any cross-section of the beam perpendicular to its axes retains its shape;
- for the external plies the classical assumption that plane cross sections remain plane and perpendicular to the longitudinal axis holds (Euler-Bernoulli assumption), whereas the interlayer has negligible axial and bending stiffness, and it can only transfer shear stresses between the external plies;
- the bonding between the plies is assumed to be perfect (no slip occurs) and the radial deformation of the plies is negligible;
- the beam curvature is constant and moderate, i.e., the thickness of the beam is infinitesimal with respect to the radius of curvature.

2.1. The mathematical model

The geometric parameters for the problem at hand are represented in Fig. 1a, where the uniformly curved sandwich beam has width b and is composed of two external layers labeled as "1" and "2", with same thickness h and Young's modulus E, connected by a thin compliant interlayer of thickness t and shear modulus G. The cross sectional area and the moment of inertia of each one of the external plies are

$$A = bh, \quad I = \frac{bh^3}{12}. \tag{2.1}$$

Let R represent the (constant) radius of curvature of the middle surface of the interlayer, coinciding with the middle surface of the whole laminated package. The curvature is considered to be moderate, i.e., the order parameter h/R is a quantity infinitesimal of the first order. An angular variable θ is introduced as shown in Fig. 1a, positive if clockwise.

The beam is subjected to a radial load *per* unit length $p(\theta)$, positive if directed towards the center of curvature, and by a tangential load $q(\theta)$, positive if pointing in the direction of increasing abscissas. Following [47], assume that $q(\theta)$ is distributed