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Dissipation inequality-based periodic homogenization of wavy materials

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ABSTRACT

In this paper we present an internal variable-based homogenization of a composite made of wavy elastic-perfectly plastic layers. In the context of a strain-driven process, the macrostress and the effective yield surface are expressed in terms of the residual stresses, which act as hardening parameters in the effective behavior of the composite. Moreover, an approximate two-steps homogenization scheme useful for composites made of matrix with wavy inclusions is proposed and a comparison with one computational and one semi-analytical homogenization method is presented.

1. Introduction

Modeling the mechanical behavior of non-linear heterogeneous materials has been the subject of many research papers from both mathematical and computational point of view [4,6,9,9–11,13,16,22,25,29,35,36,38–42,49–51]. Special attention has been paid to the case of composites with properties and/or geometry dependent on a non-linear periodicity function [5,7,8,14,15], and/or non-linear constitutive behavior of the constituent materials [14,15,52,53]. For general plasticity equations and viscoplasticity with non-linear hardening we refer to the book [1]; which covers constitutive equations of "monotone type", and to [2].

The role of dissipation inequality in homogenization of dissipative materials is crucial: it needs to be considered in both micro- and macro-level in order to lead to the correct constitutive evolution equations relating stress and internal variables. The local problem in generalized standard materials (*GSM*) was completely described by Refs. [30–34,43,47] (see also [17,18,20], based on the fundamental works of [21] and [37]. Generalized materials are described by state and internal variables. Generalized forces are then defined from the free energy function expression in terms of the above variables. Additionally, the dissipation inequality holds and by the Lagrange multiplier's technique gives the evolutionary equations.

Wavy architectures can be found in the nature or constructed for functional purposes or accidently obtained in manufacturing processes and the thermomechanical behavior of forming materials or structures under specific loading and environmental conditions is of special technological interest [23–26,54]. Wavy multilayer materials and structural components are characterized by a wavy periodicity at several scales: corrugated cross sections used to stiffen structural panels, laminated composite plates exhibiting manufacturing induced waviness with problematic behavior under compression, microstructures with wavy architectures, biological tissues such as chordea tendenea found in heart valves, where stiff collagen fibril crimp patterns control the opening and closing of the valve leaflets [24], continue to form the subject of intense research effort [22,25–28,45,54–56]. In nanotechnology, wavy interfacial morphology can enhance the overall properties of composites made of thin metallic and ceramic multilayers for magnetic, optoelectronic and high-speed electronic applications [24]. Novel fuzzy fiber reinforced composites are composed of carbon fibers, wavy carbon nanotubes and epoxy matrix, with the carbon fibers radially grown on the circumferential surfaces of the carbon fibers [14,15,28].

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The objective of this paper is twofold: first, to present an analytical dissipation inequality-based homogenization scheme for a wavy multilayered medium made of elastic-perfectly plastic components; second, to propose an approximate two-steps homogenization for a composite made of a matrix with wavy inclusions. In Section 2 a review on the role of dissipation inequality in the homogenization process of rate-independent dissipative materials is presented. More specifically, it is verified that the overall behavior of a heterogeneous material is a generalized standard material behavior and the strain-driven localization problem is formulated. Moreover, the fundamental assumptions, of additivity for the free energy and of dependence of its effective value on the microstrain and a finite number of micro-internal parameters, allow for defining the effective generalized forces through the variation of the effective energy, and subsequently of expressing the overall dissipation starting from the microstresses and the rates of internal micro-hardening "forces" and microplastic strains, in correlation with micro-yield surface. In Section 3, the analytical homogenization of a wavy layered composite made of two elastic-perfectly plastic materials is presented. This includes the analytical expressions for the effective constitutive law, for the macroscopic yield surface and for the residual microstresses in terms of the macrostrain and the plastic microstrains. An interesting finding is that, as expected [49], even if the constituents are isotropic and without hardening, the composite exhibits anisotropy and hardening due to the presence of residual stresses in the effective yield surface. Finally, in Section 4, a two-steps approximate homogenization scheme is presented for a composite with wavy inclusions and numerical examples of the proposed homogenization scheme are presented, corresponding to a unit cell under monotone and cyclic loading respectively. Moreover, the results are compared to one semi-analytical and one computational (Finite Volume Direct Averaging Micromechanics-*FVDAM* [41], method. The construction of the effective yield surface completes the numerical experiments. In three appendices, all matrices needed for the analytical expressions of micro-and macrovariables are presented.

2. The dissipation in heterogeneous generalized standard materials

We consider three spatial variables that coexist for the description of the problem. The first one is the macroscale denoted by[1] $\boldsymbol{x}$ in the heterogeneous body, at which the heterogeneities, characterized by ε, are very small compared to the whole structure and can be considered as invisible. The second spatial variable is the microscale denoted by $\frac{\boldsymbol{x}}{\varepsilon}$, which is the scale for the heterogeneities (Fig. 1). The third spatial variable is used only if the body exhibits a generalized (non-linear) periodicity.

The case of materials with generalized periodicity is of special interest for two reasons: first, since it corresponds sometimes to a non-repetitive geometry as in composites with cylindrical periodicity and second, it uses simpler unit cells and may allow semi-analytical homogenization methods [53]. The choice of the representative volume element is made with respect to the generalized periodicity vector function $\varrho(\boldsymbol{x})$ and $Y = [0,y_1] \times [0,y_2] \times [0,y_3]$ is chosen to be the basic cell, where

$$\boldsymbol{y} = \frac{\varrho(\boldsymbol{x})}{\varepsilon}. \tag{2.1}$$

[1] In the sequel, every vector or tensor will be denoted with two ways: a bold-symbol or its indicial notation. The scalar quantities appear in regular fonts.

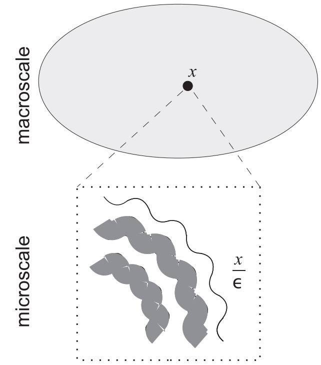

Fig. 1. Macro- and microscale.

The dependence of functions on the microcoordinate is performed (in a non-periodic way, except if $\varrho(\boldsymbol{x}) = \boldsymbol{x}$) via

$$\bar{\boldsymbol{y}} = \frac{\boldsymbol{x}}{\varepsilon}. \tag{2.2}$$

In this paper we focus our attention on the multilayered materials (see Fig. 2). For simplicity, we present the case of structures with layers parallel to the x_3-axis. Thus, at every macropoint (x_1,x_2) microstress and microstrain are uniform in every phase with values depending on (x_1,x_2) [52]. More specifically, the angle $\theta(x_1,x_2)$ of the tangent at the macropoint with x_1-axis enters the equations of microstress equilibrium and the equations of continuity at the interfaces, as well as the effective tangent modulus at (x_1,x_2).

Let us now denote field variables $\boldsymbol{\sigma}^0$, $\boldsymbol{\varepsilon}^0$ and $\boldsymbol{u}^1$ as microscopic variables and $\boldsymbol{\Sigma}$, $\boldsymbol{E}$ and $\boldsymbol{u}^0$ as the macroscopic variables. The macroscopic quantities depend only on the macrocoordinate $\boldsymbol{x}$. It is worth noticing that both classes of deformation fields are related to the representative volume element located at $\boldsymbol{x}$. Away from the boundaries, stress and strain fields conform at the microlevel to the generalized periodicity conditions:

$$\boldsymbol{\sigma}^0, \boldsymbol{\varepsilon}^0 \text{ are } Y - \text{periodic functions of } \boldsymbol{y}. \tag{2.3}$$

The actual displacement $\boldsymbol{u}^0$ within Y located at $\boldsymbol{x}$ is assumed to be expressed as a sum of a linear and a generalized-periodic part [48,50,52].

$$u_i^0(\boldsymbol{x},\bar{\boldsymbol{y}},\boldsymbol{y}) = E_{ij}\bar{y}_j + u_i^1, \tag{2.4}$$

where

$$u_i^1 = u_i^1(\boldsymbol{x},\boldsymbol{y}), \tag{2.5}$$

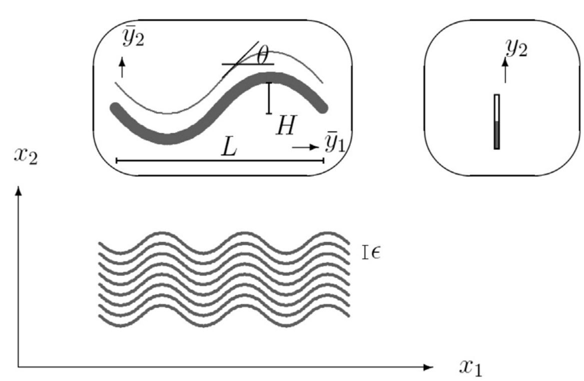

Fig. 2. Wavy multilayered material.