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Evaluation of mixed theories for laminated plates through the axiomatic/asymptotic method

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ABSTRACT

This paper proposes variable kinematic, mixed theories for laminated plates built via the asymptotic/axiomatic method (AAM). This method has been recently developed and successfully applied to develop refined theories for multilayered plates and shells. The AAM evaluates the accuracy of each unknown variables of a structural model. The present paper extends the AAM to mixed theories based on the Reissner Mixed Variational Theorem (RMVT). The displacement and transverse stress fields are modeled by means of the Carrera Unified Formulation (CUF), and expansions up to the fourth-order are employed. Equivalent Single Layer (ESL) and Layer Wise (LW) schemes are adopted, and closed-form Navier-type solutions are considered.

The AAM is exploited to determine the set of active terms of a refined plate model. The inactive terms are then discarded. The effectiveness of each variable is evaluated with respect to an LW, fourth-order mixed model. Reduced models are built for different thickness ratios, stacking sequences and displacement/stress variables.

The results suggest that reduced models with significantly less unknown variables than full models can be built with no accuracies penalties. Such models are problem dependent, and full models should be preferred in the case of thick, asymmetric plates.

1. Introduction

Laminated composite and metallic plates are commonly adopted in many engineering applications, and a number of structural models have been developed over the last decades for their analysis. The solution of the 3D elasticity equations can be very expensive from a computationally standpoint, and, moreover, such solutions are usually valid only for a few geometries, material characteristics, and boundary conditions. 2D structural models are employed to analyze plates. The oldest plate model is due to Kirchhoff [1]. According to this model, transverse shear, and normal strains are assumed to be negligible with respect to the other stress and strain components. An extension of this model to multilayered structures is referred to as the Classical Lamination Theory (CLT). Further details on shell theories can be found in Ref. [2].

Refined plate models have been developed to improve the Kirchhoff model. A brief overview of some of the main techniques to develop advanced plate models for the static analysis of composite plates is presented hereinafter. In particular, the following macro categories are addressed:

- Models that account for transverse and normal shear effects.
- Layer-Wise And Zig-Zag models.
- Asymptotic approaches and the proper generalized decomposition.
- Reissner Mixed Variational Theorem (RMVT) based models.

Particular attention is paid to the latter; the main aim of this paper deals, in fact, with refined plate models based on the RMVT.

Shear effects in laminated plates can be very significant; the shear deformability in this type of plates is higher than in isotropic plates. Reissner and Mindlin [3,4] included the shear effect, and their model is known as the First Order Shear Deformation Theory (FSDT). Further refinements of the FSDT can be achieved through the Vlasov [5] and the Reddy-Vlasov model [6]. These models

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account for the homogenous conditions for the transverse shear stresses at the top and bottom plate surfaces.

Hildebrand, Reissner, and Thomas [7] developed a refined model that accounts for both the transverse shear and normal stress effects, i.e. that fulfills Koiter's recommendation [8]. Other significant contributions on laminated plate models can be found in Refs. [9–12].

Multilayered structures are transversely anisotropic, and their mechanical properties are discontinuous along the thickness. These features are responsible for transverse displacements whose slopes can rapidly change at the layer interfaces and transversely discontinuous in-plane stresses. Due to equilibrium conditions, transverse stresses must be continuous at the interfaces. The Layer-Wise (LW) approach [13–15], Zig-zag models [16,17] and mixed variational tools [18–20] have been proposed to deal with these mechanical behaviors. In the LW, each layer is seen as an independent plate and compatibility of displacement components are imposed at the interfaces. Compatibility and equilibrium conditions are then used at the interfaces to reduce the number of the unknown variables.

The plate theories mentioned above are axiomatic; the unknown variable fields are, in fact, assumed a priori, and such assumptions are based on the scientist's intuition and experience. An alternative approach is the asymptotic method in which asymptotic expansions of the unknown variables are introduced along the plate thickness. The asymptotic method provides approximate theories with known accuracy with respect to the 3D exact solution [21–24]. The influence of the expansion terms is evaluated with respect to a geometrical perturbation parameter (e.g. the thickness-to-length ratio). The asymptotic approach furnishes consistent approximations; that is, all the retained terms are those which influence the solution with the same order of magnitude as the vanishing perturbation parameter.

The Proper Generalized Decomposition (PGD) decomposes a 3D problem as the summation of a number of 1D and 2D functions [25]. PGD can be considered as a powerful tool to reduce the numerical complexity of 3D problem.

The RMVT is a mixed variational approach in which displacements and transverse stresses are the unknown variables of the structural problem. Furthermore, in an RMVT model the interlaminar continuity of transverse stresses is imposed a priori [18,26–28]. Murakami and Toledano applied the RMVT to the analysis of multilayered plates via first and higher-order theories and layer-wise schemes [19,29,30].

This paper proposes refined plate models by means of the Carrera Unified Formulation (CUF) [31,32]. According to the CUF, the displacement and stress fields of plates can be defined as arbitrary expansions of the thickness coordinate. The expansion order is a free parameter of the analysis, and it can be chosen via a convergence analysis. The governing equations are obtained through a set of fundamental nuclei whose form does not depend on either the expansion order nor the base functions. CUF models based on the RMVT have been developed over the last years [20,33–42].

The axiomatic/asymptotic method (AAM) has been recently developed for beams [43,44] and plates [45,46] in the CUF framework. The AAM investigates the effectiveness of each generalized displacement variable of a refined theory against the variation of various parameters; such as the thickness, the orthotropic ratio and the stacking sequence. The AAM leads to the definition of reduced models that have the same accuracy of the full model but that have fewer unknown variables. The best theory diagram (BTD) is an important outcome that stemmed from the use of the AAM [47]. The BTD is a diagram in which, for a given problem, the computationally cheapest structural model for a given accuracy can be read. The BTD is problem-dependent, and it can be obtained by exploiting genetic algorithms [48,49]. The most recent developments have dealt with the definition of more accurate techniques to evaluate the accuracy of the model [50,51], layer-wise plate [52] and shell [53] models.

In this work, the AAM is applied to RMVT models for the first time. Navier-like closed-form solutions are employed, and both ESL and LW models are considered. This paper is organized as follows: the geometrical relations for plates and the constitutive equations for laminated structures are presented in Section 2; the CUF is presented in Section 3; the governing equations are introduced in Section 4; the axiomatic/asymptotic technique and the BTD are introduced in Section 5; the results are given in Section 6; the conclusions are drawn in Section 7.

2. Geometrical and constitutive relations for plates

The plate geometry is shown in Fig. 1; the reference surface is Ω and its boundary is Γ. The reference system axes which lie on the reference surface Ω are denoted as x, y; z is the reference axis normal to the reference surface. The length side dimensions of the plate are indicated as a and b, and the thickness of the plate is h.

The strain components for a generic k layer are evaluated according to the linear strain-displacement relation, that is

$$\varepsilon^k = \mathbf{D}\mathbf{u}^k \tag{1}$$

where $\mathbf{D}$ is a differential operator whose components are

$$\mathbf{D} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \end{bmatrix} \tag{2}$$

Strain components are grouped into in-plane (p) and out-of-plane (n) components, that is

$$\varepsilon_p^k = \left[\varepsilon_{xx}^k \; \varepsilon_{yy}^k \; \varepsilon_{xy}^k\right]^T \quad \varepsilon_n^k = \left[\varepsilon_{xz}^k \; \varepsilon_{yz}^k \; \varepsilon_{zz}^k\right]^T \tag{3}$$

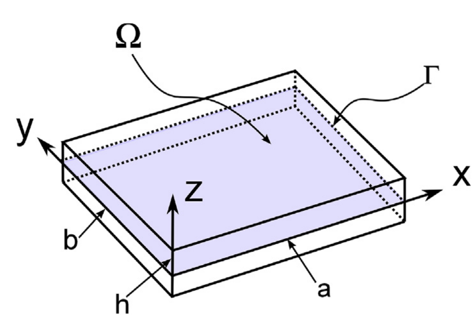

Fig. 1. Plate geometry and reference frame.