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Shear coupling effects of the core in curved sandwich beams

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ABSTRACT

We consider a composite package formed by two curved external Euler-Bernoulli beams, which sandwich an elastic core with negligible bending strength but providing the shear coupling of the external layers. This coupling considerably affects the gross response of the composite structure. There is an extensive literature on straight sandwich beams of this type, but very little attention has been paid to the effects of curvature. Here, an analytical linear elastic model is proposed for beams with arbitrary variable curvature. Equilibrium equations and boundary conditions are obtained through a variational approach. Useful simplifications are possible for the case of moderately curved beams and beams with constant curvature.

1. Introduction

Three-layered sandwich structures are commonly used in modern building, aerospace, aeronautical, automotive and naval constructions. They are composed by two external sheets and an inner core, which usually has negligible bending stiffness, but provides the shear coupling of the external layers. Optimal designs can be obtained by choosing different materials and geometric configurations of the face sheets and core. Applications in this category may range from structural insulating panels, consisting in a layer of polymeric foam sandwiched between two layers of structural board (usually sheet metal, plywood or cement), to wood elements made of layers glued together, arriving at steel beams supporting concrete slabs connected by ductile studs.

The type of sandwich that will be considered here is particular. The external layers are supposed to have noteworthy axial and bending stiffness, whereas the inner layer produces their shear-coupling. In other words, the role of the inner core is that of providing shear stresses that contribute to the gross bending stiffness of the composite package, keeping unchanged the relative distance between the external layers. Such a scheme fits to a number of cases. An example is represented by adhesive-bonded beams, where the thickness of the interlayer is so small that the variation of its height can be neglected. In general, when the outer layers are quite thick, the change in curvature due to bending remains moderate even under concentrated loads, because these are "diffused" in the softer interlayer limiting its strain in transversal direction. The required properties can also be obtained with anisotropic cores, for which the elastic stiffness in the out-of-plane direction is much higher than the in-plane stiffness: an example is honeycomb cores, which may be considered rigid in the out-of-plane direction and flexible at right angle to that.

The modeling of composite laminated structures with a "soft" core is one of the most active research fields of the last decades, since an accurate stress analysis is required to design structural parts. Hence, several theories have been developed to describe the structural behavior of sandwich beams [1,2]. In particular, the well-know "First-Order Shear Deformation" approach [3], based on the assumption that planes normal to the midplane remain straight but not necessarily normal to it after deformation, has been followed by many authors in the last decades (see, among others, [4–6]). This theory usually provides good results in terms of maximum displacement under appropriate choice of the shear rigidity. The key role played by the interlaminar shear stress on the response of the laminate composite was pointed out since the Sixties, thanks to the contributions by Pagano [7–9] and Reddy [10,11].

The effect on the deformation of the out-of-plane strain of the interlayer can certainly be of importance and, indeed, it has been a subject of recent research (see Ref. [12] and the list of references therein reported). However, if one assumes that the thickness of the interlayer remains unaltered, the problem is greatly simplified and reduces to the assessment of the degree of shear coupling offered by the inner core. The first analytical model for a composite beam with shear interaction is commonly attributed to Newmark *et al.*

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[13], who investigated the response of composite steel-concrete beams connected by elastic studs. Since then, several studies [14,1,2,15] have analyzed the effects of the interlayer properties for different compositions of the laminated package and geometries.

Although the model problem considered here is general, it can be conveniently specialized to the case of laminated glass [16]. This is a composite structure manufactured by bonding two or more thick layers of glass together with thin layers of polymer, to create a single composite sheet widely used in architecture thanks to its transparency, strength and safe post-glass-breakage performance [17]. The polymeric interlayers are certainly too soft and thin to present flexural stiffness *per se*, but they can provide shear stresses that produce the glass plies interaction [18]. Moreover, since the interlayer is much thinner that the glass plies, the variation of its thickness due to transversal actions is negligible. Of course, the degree of shear coupling depends upon the shear stiffness of the interlayer [19]. The problem has been considered by many authors, both from the experimental (see, among other [20,17,21]) and the analytical point of view. Important contributions concerning laminated glass beams can be found in Refs. [22–27], while other works have focused on laminated glass plates [28–35]. Particular attention has been paid to the buckling behavior of laminated glass (see, for example, [36–40]) and to hybrid laminated glass units [41,42].

All the aforementioned references consider flat laminated glass, but in recent years there has been a deep interested in curved elements due to the developing, as an architectural trend, of the use of free form design for curved transparent façades and roofs. Curved sandwich panels are also used in aerospace engineering and e.g., for inlet cowl panels, fuselage glove of space shuttle orbiter, certain landing gear doors [43,44]. Several studies, mainly dealing with the numerical aspects of the modelling, have been performed since the Nineties [45–47]. Other important contributions, which consider curved laminated beams composed by perfectly bonded plies with considerable bending stiffness, have been published in recent years [48–50]. A fundamental contribution has certainly been the article by Frostig [51], in which an analytical model is presented for the case of three-layered beams, with constant curvature, composed by two thin external layers, with negligible thickness (with respect to the radius of curvature) bonded by a thick and soft transversely-flexible core. The case considered here is somehow dual, because it deals with sandwich curved beams with thick external plies bonded by a transversely-stiff and shear-resistant core, which is supposed to maintain unaltered its thickness during the deformation. To our knowledge, similar problems have been considered previously only in Ref. [52], but with an approximate description of the kinematics of deformation of the layered beam and under the hypothesis that the curvature is constant and the loads are purely radial.

The plan of the article is the following. First, in Section 2, the kinematics of monolithic curved beams is briefly recalled, while in Section 3 the equilibrium equations for the composite package are derived through energy minimization. From the general treatment, we show how various hypotheses about the curvature can lead to noteworthy simplifications. In fact, in Section 4, the governing equations are specialized to some specific conditions of practical interest, such as the case of constant curvature and the case of moderate *arbitrary* curvature, i.e., the case in which the thickness of the beam is infinitesimal with respect to the radius of curvature.

2. Review of curved beam theory

Before dealing with curved layered beams, it is convenient to briefly recall the basic equations for a monolith. Many articles deal with two-dimensional curved beams [53,54,47,55], focusing on particular aspects such as, for example, the vibrations of curved beams of arbitrary shape lying in a plane [56–58] or the response of thin-walled curved beams of open section, including buckling and large displacement [59–61]. Other contributions discuss the deformation of arbitrarily curved and twisted three-dimensional curved beams, both from the analytical [62–64] and the numerical [65,66] point of view. For the case at hand, let the curve Γ, plotted with dashed line in Fig. 1, represent the reference fiber of a *plane* curved beam, i.e., a beam with undeformed axis lying on a plane, usually associated with the locus of the centroids of its cross sections but here considered arbitrary. Let s represent a curvilinear abscissa parameterized by arc length, and introduce the local reference system identified by the unit-vectors $\mathbf{t}(s)$ and $\mathbf{n}(s)$, respectively tangent and normal to Γ, with $\mathbf{t}(s)$ oriented in the direction of increasing s and $\mathbf{n}(s)$ pointing towards the center of curvature. The Frenet-Serret's equations [67,68], describing the movement of a frame system along the axis through the tangent, normal and binormal vectors, may be used to describe the kinematics of curved beams [64]. For the case of a plane beam one has

$$\frac{d\mathbf{n}(s)}{ds} = -\frac{\mathbf{t}(s)}{R(s)}, \quad \frac{d\mathbf{t}(s)}{ds} = \frac{\mathbf{n}(s)}{R(s)}, \tag{2.1}$$

where $R(s)$ is the radius of curvature of Γ at s. Any point $\mathbf{x}$ of the beam is identified by the pair (s, r), being r the distance from Γ in the direction of $\mathbf{n}(s)$.

2.1. Displacement and strain fields

The in-plane displacement of $\mathbf{x} = (s, r)$ can be expressed in the form

$$\mathbf{u}(s,r) = u(s,r)\mathbf{t}(s) + v(s,r)\mathbf{n}(s), \tag{2.2}$$

and, recalling Frenet's formulas (2.1), the strain tensor of a point of coordinates $(s, 0)$ can be written as

$$\begin{aligned}\boldsymbol{\varepsilon}(s,0) &= \frac{\nabla\mathbf{u}(s,0) + \nabla\mathbf{u}^T(s,0)}{2} \\ &= \left[u_{,s}(s,0) - \frac{v(s,0)}{R(s)}\right]\mathbf{t}\otimes\mathbf{t} + \left[\frac{u(s,0)}{R(s)} + v_{,s}(s,0) + u_{,r}(s,0)\right] \\ &\quad \times \frac{\mathbf{n}\otimes\mathbf{t} + \mathbf{t}\otimes\mathbf{n}}{2} + v_{,r}(s,0)\mathbf{n}\otimes\mathbf{n},\end{aligned} \tag{2.3}$$

where comma denotes partial differentiation with respect to the indicated variable. A more accurate description of the displacement and strain fields for planar curved beams may be found in Refs. [69,55].

Identify now an orthogonal curvilinear reference system on the beam, composed by a family of lines normal to Γ, coinciding with

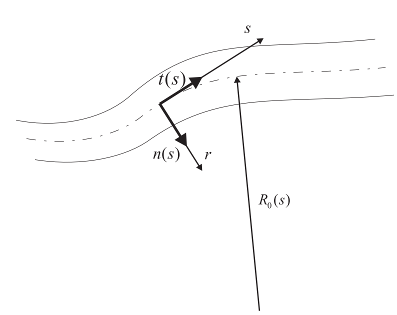

Fig. 1. Longitudinal fiber and local reference system for a plane curved beam.