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Analytical model of SiPM time resolution and order statistics with crosstalk

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ABSTRACT

Time resolution is the most important parameter of photon detectors in a wide range of time-of-flight and time correlation applications within the areas of high energy physics, medical imaging, and others. Silicon photomultipliers (SiPM) have been initially recognized as perfect photon-number-resolving detectors; now they also provide outstanding results in the scintillator timing resolution. However, crosstalk and afterpulsing introduce false secondary non-Poissonian events, and SiPM time resolution models are experiencing significant difficulties with that.

This study presents an attempt to develop an analytical model of the timing resolution of an SiPM taking into account statistics of secondary events resulting from a crosstalk. Two approaches have been utilized to derive an analytical expression for time resolution: the first one based on statistics of independent identically distributed detection event times and the second one based on order statistics of these times. The first approach is found to be more straightforward and "analytical-friendly" to model analog SiPMs. Comparisons of coincidence resolving times predicted by the model with the known experimental results from a LYSO:Ce scintillator and a Hamamatsu MPPC are presented.

1. Introduction

In the last decade silicon photomultipliers have demonstrated rapid and significant progress in becoming detectors of choice for a wide range of low light level applications. Initially recognized as a proportional detector, competitive with PMT and APD due to superior photon number resolution, SiPM has also appeared to be a rapidly emerging time-of-flight detector, first of all for Cherenkov light and short scintillation pulses [1–4].

One of the most demanded applications of SiPMs is the detection of 511 keV photons produced by LSO-based scintillator crystals due to high practical interest in the development of a new generation of TOF PET scanners. This application appears to be in a proper alignment with the key advantages of SiPM technology as it requires both good energy and time resolution of short light pulses. A remarkable progress has been seen in the time resolution of LSO scintillation detection from 1.5 ns FWHM in 2005 [5] to 108 ps FWHM [6] and now targeting to be below 100 ps [7,8]. It should be noted that in the same time period, single photon time resolution (SPTR) of SiPM has not been considerably improved upon, namely a value of 123 ps FWHM was reported for the MEPhI/Pulsar SiPM in 2003 [1] vs. 120 ps FWHM reported for the AdvanSiD SiPM—the best one from a representative set of modern SiPMs in 2012 [9] of the same area $1 \times 1\ mm^2$. Indeed, in a recent study of fundamental limits of scintillation detectors, a photoelectron detection rate (a mean number of photoelectrons N_{pe} per a scintillation decay time τ_d, i.e. N_{pe}/τ_d) is found to be a much more influential factor affecting time resolution than an SPTR, optical transport jitter, and single electron response (SER) rise time [10].

However, the results and conclusions of this study have inherent limitations in clarity of observed causal relationships and its generality because it is based on Monte Carlo simulations as well as many others in this area. Obviously, looking for the best tool for such studies, it makes sense to try to establish an analytical model of SiPM response and statistics.

SiPM response is a very specific and much more complicated process to analytically model than that of conventional PMT, APD, and PIN detectors typically used in TOF applications. High crosstalk and afterpulsing, limited number of pixels and relatively long pixel recovery time results in non-Poissonian statistics and non-linearity of the SiPM response. TOF techniques utilise as small triggering threshold as possible and are focused on the early phase of SiPM response acquisition (from a single to tens of photoelectrons arriving in a sub-nanosecond time scale), a non-linearity is assumed to be

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negligible as well as an afterpulsing effect. In contrast, crosstalk generates false secondary single electron pulses with considerable probability (up to 50% at high overvoltages used to be sought-after for the best time resolution) and almost instantly with the primary ones, and it should be properly taken into account in any model.

However, most analytical models experience significant difficulties in accounting for multiple crosstalk events. For example, rather comprehensive models of SiPM time resolution are limited by assumptions either that only one crosstalk event could be produced by a single primary event [11] or that the primary fired cell could initiate crosstalk events in no more than in three neighbour cells aside from the primary one [12].

At the same time, an analytical approach has been found to be a powerful tool to derive relatively simple and realistic models of crosstalk and afterpulsing processes as a geometric distribution chain and branching Poisson processes [13,14] as well as to include these results into analytical models of SiPM response statistics and photon number resolution using a total excess noise factor approach [15]. Initial thoughts regarding possibility to derive an analytical expression of SiPM time resolution combining analytical results on the total excess noise factor of SiPM and on the mean SiPM response dynamics have also been recently discussed [16].

Therefore, the motivation of this study is to try to move as far as possible into advanced analytical modelling of SiPM time resolution with crosstalk utilizing in some extent previous results pointed above.

2. Method

2.1. Approach based on statistics of independent detection times

Modelling of time resolution of scintillation detectors with photomultiplier readout has been in active progress since the 1950s due to its high practical importance [17–19].

According to the well-known approach, a mean μ_{out} and standard deviation σ_{out} of photomultiplier output response with some external electronic noise σ_e has to be derived, and, in the case of the leading edge discriminator technique a threshold D is used for the output pulse detection, the time resolution σ_t is estimated in a threshold crossing time T_D as

$$\sigma_t = \left.\frac{\sqrt{{\sigma_{out}}^2 + \sigma_e^2}}{d\mu_{out}/dt}\right|_{\mu_{out}(t=T_D)=D}. \tag{1}$$

Applying probability theory of random processes to detection of a photon signal by a photomultiplier, the output response could be considered as the filtering of a marked point process of detected events (photons) by the instrumental response function (IRF i.e. SER pulse) [20].

A point process is a series of events represented by Dirac delta functions $\delta(t-t_i)$ where event times t_i ($i=1,\ldots,N$) are independent identically distributed (i.i.d.) random variables and N is a random number of events in a given time interval (Fig. 1a). Intensity of a point process $\lambda(t)$ is defined as a mean event rate in an infinitesimal time interval at time t. Marked (often called amplified) point process $X_{in}(t)$ is an advanced consideration of a point process with i.i.d. random amplitudes (marks or amplifications) A_i *of* events (Fig. 1b).

If every event i of a marked point process produces an output response of non-random temporal shape and with an amplitude proportional to A_i then an output response is a random function $Y_{out}(t)$ with some mean and variance, and it is considered as a result of filtering of a marked point process by the instrumental response function $h(t)$ (Fig. 1c). Filtering represents a convolution of the input and response functions. For example, the most common case of filtered marked Poisson point process is described as the following:

$$X_{in}(t) = \sum_{i=1}^{N} A_i \times \delta(t-t_i) Y_{out}(t) = \sum_{i=1}^{N} A_i \times h(t-t_i)$$

$$E[Y_{out}(t)] = E[X_{in}] * E[h] = \overline{A} \times [\lambda * h](t) \overline{N} = \int_0^\infty \lambda(t')dt'$$

$$\mathrm{Var}[Y_{out}(t)] = \mathrm{COV}[X_{in}] * \mathrm{COV}[h](t-t')\big|_{t'=t}$$

$$= \overline{A}^2 \times \left(1 + \frac{\mathrm{Var}[A]}{\overline{A}^2}\right) \times \left[\lambda * h^2\right](t). \tag{2}$$

where the asterisk sign ($*$) is used to define a convolution of adjacent functions.

An expression for the variance (2) is well-known as the Campbell theorem.

Adapting (2) to light pulse detection by SiPM, it is convenient to substitute some denotation and rewrite these results as follows:

$$\lambda(t) = \overline{N_{pe}} \times [\rho_{ph} * \rho_{sptr}](t)$$

$$E[V_{out}(t)] = \overline{V}_{ser} \times [\lambda * h](t)$$

$$\mathrm{Var}[V_{out}(t)] = \overline{V}_{ser}^2 \times ENF_{gain} \times \left[\lambda * h^2\right](t), \tag{3}$$

where N_{pe} is a Poisson distributed number of photoelectrons per light pulse, ρ_{ph} is a probability density function of photon arrival times t_i, ρ_{sptr} is a probability density function of a SiPM jitter (SPTR), $V_{out}(t)$ is an output response voltage, V_{ser} is an amplitude of SER pulse, and ENF_{gain} is an excess noise factor of SiPM gain.

However, being based on a Poisson point process, this model provides rather simple expressions (3), which could be relevant for APD and PMT (with low afterpulsing), but it does not consider non-Poissonian effects.

2.2. Advances in non-Poisson point process with correlated events

Looking for a chance to take into account non-Poissonian effects, let us turn to an advanced theory of random processes.

The main characteristics of non-Poissonian point processes $X_{in\text{-}n}(t)$ have been well studied and analytically expressed with respect to the specific cases of our interest [21].

The first case is related to the randomness in an intensity function $\lambda(t)$ which corresponds to a double stochastic Poisson distribution of generated photons. It allows accounting for the intrinsic scintillator resolution δ_{sci} as an excess variance component atop of the Poisson variance with the same mean number of photons:

$$\mathrm{Var}[N] = \overline{N} + \delta_{sci}^2 \times \overline{N}^2. \tag{4}$$

The second case is much more complicated as it concerns the secondary correlated events (often called cascaded or clustered) produced by the primary ones. Each primary event i is to produce a random number of secondary events K_i, and each jth secondary event has an i.i.d. random displacement time Δt_{ij} from the primary event time t_i as shown on Fig. 1d (dashed line). Thus, the total point process is expressed as a sum of primary and secondary events:

$$X_{in-n}(t) = \sum_{i=1}^{N} \sum_{j=0}^{K_i} A_{i,j} \times \delta(t - t_i - \Delta t_{ij}). \tag{5}$$

Using known results of the mean and covariance of $X_{in\text{-}n}(t)$ [21], assuming that IRF of crosstalk events is just the same $h(t)$ as for any primary event, and applying a filtering procedure to the correlated point process (5), the SiPM output response can be expressed as

$$E[V_{out}(t)] = \overline{V}_{ser} \times \overline{N}_{pe} \times (1 + \overline{K}_{sec}) \times [\rho_{tot} * h](t)$$