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A quantum mechanical analysis of Smith–Purcell free-electron lasers

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ABSTRACT

The paper presents a quantum mechanical treatment for analyzing the Smith–Purcell radiation generated by charged particles passing over a periodic conducting structure. In our theoretical model, the electrons interact with a surface harmonic wave excited near the diffraction grating when the electron velocity is almost equal to the phase velocity of the surface wave. Then, the surface harmonic wave is electromagnetically coupled to a radiation mode. The dynamics of electrons are analyzed quantum mechanically where the electron is represented as a traveling electron wave with a finite spreading length. The conversion of the surface wave into a propagating mode is analyzed using the classical Maxwell's equations. In the small-signal gain regime, closed-form expressions for the contributions of the stimulated and spontaneous emissions to the evolution of the surface wave are derived. The inclusion of the spreading length of the electron wave to the emission spectral line is investigated. Finally, we compare our results based on the quantum mechanical description of electron and those based on the classical approach where a good agreement is confirmed.

1. Introduction

In 1953, S.J. Smith and E.M. Purcell firstly demonstrated that an optical light is emitted when an electron beam moves parallel and close over a metallic diffraction grating in vacuum [1]. The process was understood in terms of a simple model based on oscillations of the image charges induced on the metallic surface by electrons. The Smith–Purcell (SP) effect is widely considered as a possible mechanism for free-electron laser (FEL) operating over a wavelength range extending from the millimeter [2,3] to the optical region [4,5]. In recent experiments [6–8], it has been shown that the SP radiation provide a promising candidate to realize compact and tunable radiation source in the THz region. FELs based on the SP effect are operated in the amplifier and oscillator configurations where the optical feedback is required in the latter case [9–14]. Such FELs could be made with much more compact device structure compared to other FELs (e.g., undulator FELs), and therefore may be interesting for application.

Many theoretical analyses have been developed to analyze the dynamic of the Smith–Purcell radiation [15–22]. In most of these analyses, the metallic waveguide with periodic corrugation behaves as a slow wave structure through which a slow space harmonic of a transverse magnetic (TM) Floquet mode is propagated. The TM mode has a longitudinal electric field component and interacts most strongly with the traveling electrons. The working principle involves the synchronism between the velocity of the electron beam and the phase velocity of the TM surface modes of the periodic structure. The SP radiation contains a broad continuous frequency band and the radiation wavelength is determined by the observed angle, period of grating, and electron beam energy. Since the minimum corrugation period can be obtained currently is $\lesssim 0.1\ \mu m$, the generation of SP radiation operating up to the ultraviolet is commonly accomplished by using nonrelativistic electron beams. In these cases, the radiation wavelength is a fraction of the corrugation period.

In all forgoing analyses, the interacting electron is consider as a point particle where its spreading size is assumed to be much shorter than the period of grating as well as the wavelength of emitted radiation. M. Yamada in [23] developed a quantum mechanical treatment for calculating the amplification gain of an optical amplifier in which an electron beam passes above a dielectric planer waveguide. This amplifier is basically one type of the Cherenkov FELs. The theoretical analysis was performed based on the density matrix formalism which is a quantum statistical treatment [24–26]. In the theoretical model of [23,27,28], the electron is represented by an electron wave with a finite spreading length. In [27,28], the validity of the theoretical model is examined by comparing the experimentally measured data of intensities and spectrum profiles of optical radiation with those predicted theoretically. It is confirmed that the spectral profile of emission is characterized by the spreading length of the electron wave.

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In this paper, we present a new theoretical analysis for the SP radiation in the small-signal low gain regime. In our analysis, the dynamic of the EM wave is described using the Maxwell's wave equations. On the other hand, the dynamic of electrons are quantum mechanically analyzed using the density matrix formalism. We derive a generalized expression for the dispersion function that determines the spectral profile of emitted radiation. In this expression, the inclusion of the finite spreading length of the electron wave is studied. Our analysis is devoted for the non-relativistic electron energies (≤ 100 keV). Also, since low-density beams are utilized in the SP experiments constructed to date, the space charge effects are neglected.

In Section 2, the analytical representations of the EM waves are shown. The EM wave is assumed to compose of surface modes that propagate along the corrugated surface and radiation modes that are emitted from the corrugated surface. Formulations of the radiation power and the stored energy are presented. In Section 3, the electron dynamics are described on the basis of the density matrix method. In Section 4, the gain coefficient of amplification by the stimulated emission and the radiation rate by the spontaneous emission are derived. In Section 5, the resonance condition for beam-radiation interaction is introduced, and that the spectrum characteristics of SP radiation are discussed in details. In Section 6, we present a comparison between our results and those obtained in a well-known classical analysis where a satisfactory agreement is reached. Finally, conclusions are given.

2. Analytical model and formulation of optical wave

2.1. Representation of optical wave

An illustration of the SP effect is shown in Fig. 1 where an electron beam moves at a distance h above a metallic corrugated surface with spatial periodicity Λ in the z direction. The depth direction of the corrugation is y and the width direction is x. The corrugation is uniform and oriented in the x direction. We also assume that that cross-sectional area of the electron beam is $w \times w$ in the $x-y$ plane.

Variations of the electric field $\boldsymbol{E}(x,y,z,t)$ and the magnetic field $\boldsymbol{H}(x,y,z,t)$ of the EM wave are given by Maxwell's wave equations as

$$\nabla^2 \boldsymbol{E} - \mu_0 \sigma(x,y,z)\frac{\partial \boldsymbol{E}}{\partial t} - \mu_0 \varepsilon(x,y,z)\frac{\partial^2 \boldsymbol{E}}{\partial t^2} = \mu_0 \frac{\partial \boldsymbol{J}}{\partial t} \tag{1}$$

$$\nabla^2 \boldsymbol{H} - \mu_0 \sigma(x,y,z)\frac{\partial \boldsymbol{H}}{\partial t} - \mu_0 \varepsilon(x,y,z)\frac{\partial^2 \boldsymbol{H}}{\partial t^2} = -\nabla \times \boldsymbol{J} \tag{2}$$

where $\sigma(x,y,z)$ and $\varepsilon(x,y,z)$ are the electrical conductivity and dielectric constant, respectively, which have different values in the metal and vacuum regions. $\boldsymbol{J}(x,y,z,t)$ is the current density of the electron beam where the moving electrons produces an EM wave on the grating. The interaction between the EM field localized close to the grating surface and the electron beam to obtain radiation power is counted through this current density $\boldsymbol{J}$. The evaluation of $\boldsymbol{J}$ is performed with the help of quantum mechanical treatment assuming the electron wave representation as will be given in following sections. In our model, it is assumed that a sufficiently intense magnetostatic field in the direction of the beam flow is applied. Then, the electron beam is considered to be thin where the transverse velocities of electrons in the direction normal to the electrons propagation can be neglected. In the limit of a thin electron beam, we can neglect the effects of the self-magnetic fields in the transverse directions on the longitudinal modulations of electrons. Therefore, the EM wave is restricted to be a TM mode having H_x, E_y and E_z components where the z-component of the electric field E_z interacts most strongly with the electrons.

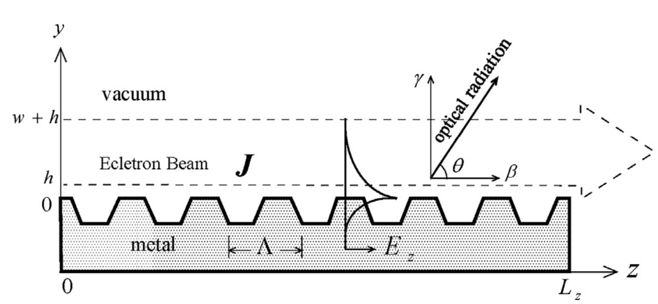

Fig. 1. The configuration of the Smith–Purcell FEL. The electron beam moves above the grating surface in the z direction. The grooves repeat periodically with the grating period Λ, the grating surface at $y=0$, and the system is invariant in the x direction.

We define the spatial wavenumber of the corrugation corresponding to the grating period Λ as

$$G = \frac{2\pi}{\Lambda} \tag{3}$$

Here, we assume the position of the metal surface in the vertical direction y is varied as

$$y_0 = \sum_{m \geq 0} d_m \cos(mGz) = \sum_{m \geq 0} \frac{d_m}{2}\left(e^{jmGz} + e^{-jmGz}\right). \tag{4}$$

In Eq. (4), m is an integer and the Fourier coefficients d_m determine the peak-to-peak depth of the grating whereas d_0 refers to the average depth of the periodic structure. As shown in Fig. 1, we set $y=0$ at the top boundary of the corrugation, the interaction length is L_z, and the distribution width of the EM wave is L_x. A part of the energy associated with the surface waves is transformed as radiation waves. The wavenumbers of both types of waves along z direction are modified by the corrugation, and then the fields' distribution is critically modified by the corrugation.

The magnetic field component H_x can be written as

$$H_x(x,y,z,t) = H_r(x,y,z,t) + H_s(x,y,z,t). \tag{5}$$

In Eq. (5), H_r is a radiation wave component and is given by

$$H_r = \tilde{E}(t)\sqrt{\frac{\varepsilon_0}{\mu_0}}\left\{R^{(+)}e^{-j\gamma y - j\beta z} + R^{(-)}e^{-j\gamma y + j\beta z}\right\}e^{j\omega t} + c.c. \tag{6}$$

and H_s is a surface field expressed as a superposition of plane waves of different frequencies as

$$H_s = \tilde{E}(t)\sqrt{\frac{\varepsilon_0}{\mu_0}}U_x(x,y,z)e^{j\omega t} + c.c. \tag{7}$$

where

$$U_x(x,y,z) = \sum_{m=-\infty}^{m=\infty}\left\{A_m^{(+)}(y,z)e^{-j(mG+\beta)z} + A_m^{(-)}(y,z)e^{-j(mG-\beta)z}\right\}. \tag{8}$$

where, ω is the angular frequency, $\tilde{E}(t)$ is the temporal field amplitude and $c.c.$ indicates the complex conjugate of the preceding terms. β and γ are the wavenumbers (i.e., propagation constants) in the z and y directions of the radiation wave, respectively. Superscripts $(+)$ and $(-)$ indicate the forward and backward propagating components along z direction, respectively. $R^{(\pm)}$ and $A_m^{(\pm)}$ are amplitude coefficients and amplitude functions, respectively.

The radiation field H_r exists only in the vacuum region where $y \geq 0$. By putting $\varepsilon = \varepsilon_0$, $\sigma = 0$, and $\boldsymbol{J} = 0$ in Eq. (2), the relation

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