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# Predicting evolution of ply cracks in composite laminates subjected to biaxial loading

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ABSTRACT

An energy-based model is developed to predict the evolution of sub-critical matrix crack density in symmetric multidirectional composite laminates for the case of multiaxial loading. A finite element-based numerical scheme is also developed to evaluate the critical strain energy release rate, $G_{Ic}$, associated with matrix micro-cracking, a parameter that previously required fitting with experimental data. Furthermore, the prediction scheme is improved to account for the statistical variation of $G_{Ic}$ within the material volume by using a two-parameter Weibull distribution. The variation of $G_{Ic}$ with increasing crack density is also accounted for based on reported experimental evidence. The simulated results for carbon/epoxy and glass/epoxy cross-ply laminates demonstrate the ability of the improved model to predict the evolution of multidirectional ply cracking. By integrating this damage evolution model with the synergistic damage mechanics approach for stiffness degradation, the stress-strain response of the studied laminates is predicted. Finally, biaxial stress envelopes for ply crack initiation and predetermined stiffness degradation levels are predicted to serve as representative examples of stiffness-based design and failure criterion.



## 1. Introduction

The emergence of polymeric composites as core materials for many different industrial applications has resulted in important studies aimed at predicting their long-term durability and damage tolerance capabilities. Particularly, the problem of transverse ply cracking in composite laminates has been investigated extensively during the last three decades [1–9]. Many of these studies have proposed models that are based on variational approaches [2,4], shear lag approximations [5], other stress-transfer methods [6], or explicitly on crack opening displacements [7]. However, most of the reported studies considered uniaxially loaded cross-ply laminates containing only 90° ply cracks, where a recent review on this subject was treated in Ref. [10]. This is mainly due to the limitations of some of the aforementioned models, which cannot account for variations in the laminate stacking sequence, or the scenario of simultaneous cracking in plies with different orientations. Practical applications require multidirectional laminates that can provide more comprehensive directional stiffness properties, thus recent studies are increasingly focused on damage evolution and stiffness degradation of multidirectional laminates containing ply cracks in multiple orientations [11–22]. The models reported in these studies generally have the capability to consider cracking in multiple plies, but many cannot directly account for intra-ply crack interactions (i.e., the so-called crack shielding effect) or inter-ply crack interactions that result from the constraining effects between adjacent plies in a laminate. Due to the complex nature of the damage evolution process in multidirectional laminates, available experimental data has been mainly limited to uniaxial tensile loading, allowing corresponding crack evolution models to be calibrated and validated [13, 19]. The problem becomes more complex when multidirectional laminates are subjected to multiaxial loads, representing the real application of composite structures. In these situations, the evolving damage processes and the corresponding material behaviour will undoubtedly change [23]. For example, in addition to cracking in the 90° and off-axis plies, a biaxial stress state will cause axial splitting cracks to develop in the on-axis plies as has been experimentally observed [24]. Furthermore, the evolving multidirectional ply cracks initiate and progress differently, resulting in complex three dimensional stress states in the laminate that vary with

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progressive loading. The development of an accurate physically-based analytical model that accounts for such complexities is essential for predicting the durability and damage tolerance capabilities of practical composite structures, which would enable safer and more cost effective designs.

In recent years, a number of reported studies have proposed damage evolution models for composite laminates that account for multiaxial loading [6,25–32], but these models have notable shortcomings that limit their applicability. On one hand, some use ultimate failure criteria to predict failure envelopes [25], while others use simplified strength-based approaches and may not consider the evolution of specific physical damage modes [26,28]. Due to the progressive nature of failure development in composites, it is important not only to consider damage evolution, but also to combine this with accurate stiffness predictions in a coherent fashion. Furthermore, many of these models are applicable only to laminates containing only cracks in one transverse direction [6, 27]. Additionally, some models simplify the inherent complex boundary value problem by assuming that a two dimensional geometric representation of ply cracks is sufficient [27, 28, 31]. Such a representation does not accurately capture the local crack behaviour and the surrounding stress state, and thus a three dimensional solution becomes necessary when multiple ply cracks are present [23]. A three dimensional model is necessary in order to accurately capture the constraining effect between the adjacent plies in a laminate. Due to inherent simplifications, the models cannot account for the interaction between cracks in different adjacent layers caused by the ply constraining effect [29, 30, 32], despite experimental observations to the contrary [13]. Another issue with existing models is that some do not consider the statistical variation of the crack evolution process [27–30], arising due to variations with manufacturing processes. Finally, all of the indicated models rely on extensive empirical data for calibration of the damage (or failure) parameters, often relying on fitting numerical constants to match experimental data. The predictive capabilities are therefore limited in scope and application.

The main objective of this study is to develop an approach for predicting damage evolution in multidirectional composite laminates subjected to multiaxial loading that accounts for stochasticity of the damage process. An energy-based approach for predicting crack density evolution developed by Joffe et al. [7] for cross-ply laminates, and later extended for multidirectional laminates by Singh and Talreja [19], is further improved here for the case of multiaxial loading. Improvements are also made to account for the stochastic nature of the cracking process, which is particularly important for an accurate prediction of crack initiation strain levels. The model capabilities are highlighted for a number of carbon fiber/epoxy and glass fiber/epoxy cross-ply laminates, and the predictions are verified with experimental data available in the literature. It should be noted that to the knowledge of the authors, experimental or predicted crack density evolution data for multiaxially loaded cross-ply laminates are not available in the literature; thus, a complete prediction of overall deformation behaviour of composites undergoing progressive damage under multiaxial loading has not been possible. This is resolved by integrating the energy-based damage evolution model with a synergistic damage mechanics model for predicting stiffness degradation. Such an approach combines the strengths of micromechanics and continuum mechanics, relying on computational micromechanics, in lieu of experimental testing, to calibrate the material damage parameters [18, 23]. Also, since the computational micromechanical models are three-dimensional, ply constraining effects and both intra-ply and inter-ply crack interactions are explicitly considered.

## 2. Modeling approach

### 2.1. Damage representation, stiffness degradation and laminate constitutive equations

Consider a general laminate consisting of on-axis, off-axis and transverse plies with unidirectional fibers as shown in Fig. 1. The evolution of ply cracks in multidirectional laminates subjected to

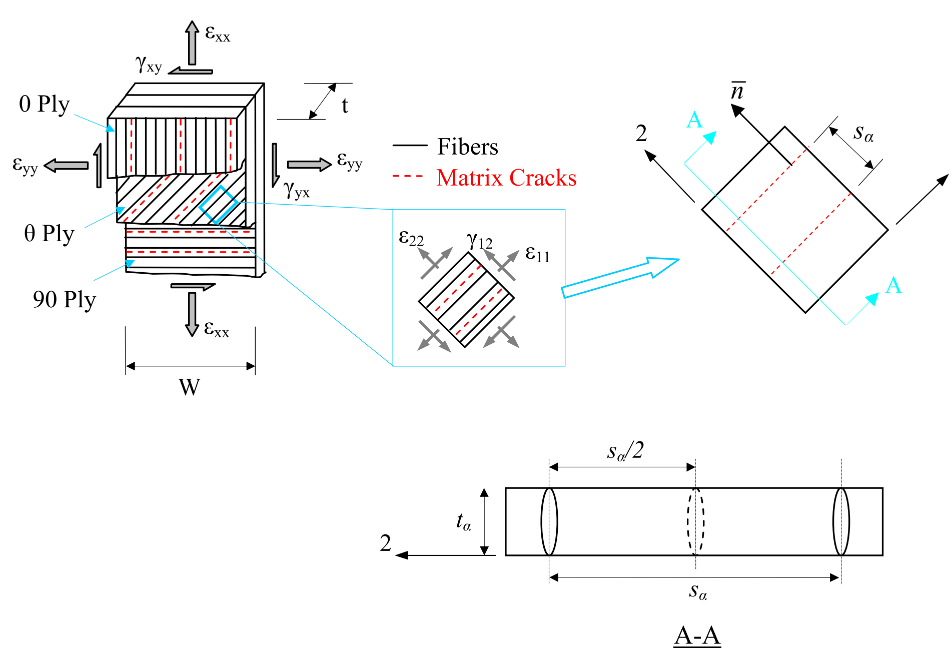


Fig. 1. A representative volume element of a damaged multidirectional laminate subjected to a 2D multiaxial strain state, with local transformed strain components and a virtual crack shown.