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Collapse load of composite laminates: Lower bound evaluation by stress field analytical approximation

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ABSTRACT

The paper proposes a limit analysis approach to define the ultimate load capacity of orthotropic composite laminates under biaxial loading and plane stress conditions. A lower bound to the collapse load multiplier is computed by solving a maximization nonlinear problem, according to the static theorem of limit analysis. To set up the optimization problem a stress field distribution is hypothesized at lamina level, moreover inter-lamina stresses are also considered. The effectiveness and validity of the proposed approach is shown by comparing the obtained numerical predictions both with available experimental data and with other numerical results carried out by means of a different numerical lower bound approach.

1. Introduction

Composite materials, due to their mechanical and physical properties, are nowadays increasingly used in many advanced engineering fields such as aeronautical, marine and civil engineering. This justifies the continuing scientific interest both in theoretical and experimental studies of such materials. Among the various problems faced on the subject, a paramount effort has been made to determine the ultimate strength of composite laminates to be used for design purposes. However, because of anisotropy and heterogeneity of composite laminates it is difficult to predict strengths. On the other hand, a failure process in a laminate can arise with different failure modes due to matrix crushing, fiber rupture, fiber buckling, delamination and/or a combination of the above phenomena [1]. Furthermore, the failure process is also influenced by the laminate lay-up, the number of layers, their orientation and stacking sequence.

Due to the complexity of the problem, several predicting failure theories for composite laminates have been developed in literature [2]. Some theories are based on linear or nonlinear analysis, some involve damage and/or fracture mechanics concepts, some are applied at lamina level others are applied on a homogenized laminate, some are based on analytical models others on numerical analysis, some others are physically based (see e.g. Refs. [3–5]). The list is not exhaustive and is out of the scope of this paper to provide an overview on the subject.

About ten different theories of predicting failure in a laminate were compared by Soden et al. [6,7], by analyzing the results of the *Word Wide Failure Exercise* (WWFE) that remains an open benchmark for those who want to validate a failure theory on composite laminates.

Besides theories presented in Ref. [2], among others, a valid alternative to predict the strength capability of a composite laminate is given by the application of nonstandard limit analysis. The nonstandard limit analysis approaches are based on the Radenkovic static and kinematic theorems of limit analysis [8] and allow to evaluate a lower and an upper bound to the collapse load multiplier of a composite laminate in a *direct manner*, i.e. without carrying out a complete post-elastic analysis of stress or strain in the laminate, so resulting relative simple methods of practical connotation for design purposes. It is worth noting that, consistently with a limit analysis theory, these approaches do not allow to describe phenomena arising after a state of incipient collapse such as delamination, debonding or damage in a wider sense.

Despite the above observation, several limit analysis approaches, in the field of composite material structures, have recently been proposed in many scientific contributions and with different scopes. Among others, some studies provided the explicit analytical form of

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the upper bound multiplier for composite plates in tension [9] or in bending [10] by minimizing the expression of an appropriately derived dissipation power. Other studies are mainly devoted to the determination of plasticity/strength domains for composites. In Ref. [11] for example, by using the homogenization method of periodic media, the plasticity domain of metal matrix composites is found by solving a limit analysis problem on a unit cell. Papers [12,13] are aimed at determining the plasticity domain of composite laminates and pin-loaded composite laminates, respectively, under a tensile loading; in these contributions the laminate is modeled as a three-dimensional solid and the problem is solved by combining finite element methods and mathematical programming numerical procedures. As an alternative approach, limit state solutions may be obtained from sequences of elastic (linear) analyses of the structure. In this case the elastic parameters of the constituent materials of the structure are suitably changed to mimic inelastic phenomena. In particular, to this kind of approach belong the Linear Matching Method (LMM) and the Elastic Compensation Method (ECM). The former is a procedure aimed at constructing a collapse mechanism for the evaluation of an upper bound, while the second is a procedure aimed to constructing an admissible stress field suitable for the evaluation of a lower bound. Both procedures have recently been applied in the field of composite materials to determine the load bearing capacity of multi-layers composite laminates and pin-loaded structural elements (see e.g. Refs. [14–19]).

Finally, a broad overview of the state of the art concerning numerical and theoretical developments of direct methods can be found in the books [21–23] and references therein, which collect the results of international workshops on this topic.

In the context of limit analysis procedures may be inserted the present paper which proposes a lower bound approach to predict the strength capability of orthotropic composite laminates under biaxial loading and plane stress conditions. In particular, the implemented procedure considers a multilayered domain which is a 3D cylindrical domain. Each layer obeys, by hypothesis, a Tsai-Wu type criterion [24] and, in particular, a second order polynomial form of it, the latter assumed as yield condition. Moreover, for each layer, a stress field distribution has been hypothesized in such a way that it satisfies boundary conditions and it results in equilibrium with the applied loads. A nonlinear optimization problem is then solved to determine the searched lower bound to the collapse load multiplier.

Four numerical examples are carried out to validate the proposed approach by comparing the obtained results both with experimental data and with the results of another limit analysis approach available in literature. Precisely, the first two examples are aimed to construct a biaxial failure envelope for two multilayered composite laminates under biaxial loads, the latter are described in detail in the well documented WWFE [6,7]. The second couple of examples concerns the peak load prediction of a square composite plate having different material configurations and subjected to two different loading conditions, in this case the obtained results are compared with those obtained by Pisano et al. via the elastic compensation method [17]. For all the examined examples, the predicted ultimate loads appear in good agreement with the expected ones so validating the proposed approach and encouraging for further investigations.

2. Limit load of a composite laminate subjected to biaxial loading – theoretical background

2.1. Geometry and mechanical model

To model the laminate consisting of n unidirectional plies, a mechanical model was developed for a multilayer domain with n anisotropic layers (Fig. 1). The laminate is designed as a cylindrical domain Ω of $\mathbb{R}^3$ with a plane base $\omega = [-a/2, a/2] \times [-b/2, b/2] \in \mathbb{R}^2$ and n layers (or plies). Each ply is designed as a cylindrical domain Ω_i with a thickness e^i. The overall thickness of the multilayer laminate is denoted by $h = \sum_{i=1}^{n} e^i$. In the following, a single bar beneath the symbols denotes a vector, two bars indicate a second order tensor. The set $(\underline{e}_x, \underline{e}_y, \underline{e}_z)$ is an orthogonal vectorial basis of Ω with $(\underline{e}_x, \underline{e}_y) \in \omega$ and $\Omega = \omega \times [0,h]$. The relative cylindrical domains for the different plies are denoted by $\Omega_i = \omega \times [\sum_{i=1}^{i-1} e^i, \sum_{i=1}^{i} e^i]$ for the layers $i \in \{2,\ldots,n\}$ and $\Omega_1 = \omega \times [0, e_1]$ for the layer 1 at the bottom of the laminate. The interface between two adjacent layers i and $i+1$ is represented by the surface $\Omega_i \cap \Omega_{i+1}$ (Fig. 1).

Looking at Fig. 1, a biaxial loading condition is assumed in which membrane loads (N_{xx},N_{yy}) are applied at the four edges of the laminate, i.e. at $x = \pm a/2$ and $y = \pm b/2$. The membrane loads are defined as:

$$
\begin{aligned}
N_{xx} &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{xx} dz = \sum_{i=1}^{n} \int_{h^{i-1}}^{h^i} \sigma_{xx}^{i} dz \\
N_{yy} &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{yy} dz = \sum_{i=1}^{n} \int_{h^{i-1}}^{h^i} \sigma_{yy}^{i} dz
\end{aligned}
\tag{1}
$$

where σ_{xx} and σ_{yy} are the stress values at the edges of the laminate $x = \pm a/2$ and $y = \pm b/2$, respectively. Similarly, σ_{xx}^i and σ_{yy}^i are the stress values at the edges $x = \pm a/2$ and $y = \pm b/2$ in a layer i and h^i indicates the ordinate of the upper face of ply i or, equivalently, of the lower face of ply $i+1$, see again Fig. 1.

2.2. Stress tensor, equilibrium equations and boundary conditions

2.2.1. Stress tensor

The second order stress tensor is written in Ω_i, the field related to ply i, as follows:

$$
\begin{aligned}
&\underline{\underline{\sigma}}^i\left(\underline{X}\right) = \left[\sigma_{\alpha\beta}^i\left(\underline{X}\right)\right] \quad \text{where} \\
&\underline{X} = (x, y, z), \quad i \in \{1 \ldots n\} \quad \text{and} \quad \alpha, \beta \in \{x, y, z\}.
\end{aligned}
\tag{2}
$$

Because of the symmetry of geometry, loading and boundary conditions, the study can be limited to a quarter of the laminate, i.e. only the domain $(x, y) \in [0, a/2] \times [0, b/2]$ is analyzed. We assume $\sigma_{xx}^i(\underline{X})$ and $\sigma_{yy}^i(\underline{X})$ to be linear functions of x and y, respectively, and satisfying the boundary conditions at the edges. The tangential stress components $\sigma_{xy}^i(\underline{X})$ are taken constant per ply i, i.e.:

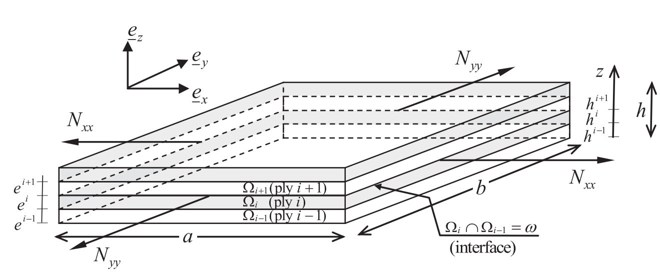

Fig. 1. Schematic mechanical model of a multilayer composite laminate subjected to a biaxial loading condition.