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Linear and nonlinear free vibration of a multilayered magneto-electro-elastic doubly-curved shell on elastic foundation

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ABSTRACT

Nonlinear and linear free vibration of symmetrically laminated magneto-electro-elastic doubly-curved thin shell resting on an elastic foundation is studied analytically. The shell is considered to be simply-supported on all edges and the magneto-electro-elastic body is poled along the z direction and subjected to electric and magnetic potentials between the upper and lower surfaces. To obtain the equations of motion, the Donnell shell theory in the presence of rotary inertia effect is used. Moreover, Gauss' laws for electrostatics and magnetostatics are used to model the electric and magnetic behavior. The nonlinear partial differential equations of motion are reduced to a single nonlinear ordinary differential equation by introducing a force function and using the single-term Galerkin method. The resulting equation is solved analytically by Lindstedt-Poincare perturbation method. After validation of the present study, several numerical studies are done to investigate the effects of foundation parameters, geometrical properties of the shell, and electric and magnetic potentials on the linear and nonlinear behavior of these smart shells.

1. Introduction

Magneto-electro-elastic (MEE) composite materials are smart materials which exhibit a coupling between mechanical, electric and magnetic fields and are able to convert energy among these three energy forms.

Static and dynamic behaviors of smart plates and shells have been studied extensively in the past years [1–8]. Many studies have been done on static and linear vibration analyses of MEE beams and plates. Pan [9] studied multilayered MEE plates analytically for the first time and derived exact solutions for three-dimensional MEE plates. Later, Pan and Heyliger [10] derived analytical solutions for free vibrations of these smart plates. The same authors [11] studied the response of multilayered MEE plates under cylindrical bending. Buchanan [12] compared the frequencies of layered MEE plates with those for the multiphase ones. Ramirez et al. [13] presented an approximate solution for the free vibration problem of two-dimensional MEE laminated plates. They [14] also determined natural frequencies of orthotropic MEE graded composite plates by using a discrete layer model. Annigeri et al. [15] studied free vibration of multiphase and layered magneto-electro-elastic beams by finite element method. A higher-order finite element model [16] and a three-dimensional model [17] have been used to study the linear vibration response of these smart composite plates, too. A closed form expression for the transverse vibration of a MEE thin plate has also been derived based on the Kirchhoff thin plate theory [18]. Davì and Milazzo [19] used a variational boundary model to study free vibration of MEE beams and bimorphs. Milazzo [20–22] and Milazzo and Orlando [23] presented single-layer approaches to static and free vibration analysis of MEE laminated plates. A one-dimensional model for the dynamic problem of MEE laminated beams was also introduced by Milazzo [24]. Recently, Chen et al. [25] studied the free vibration of multilayered MEE plates under combined clamped/free boundary conditions. Li et al. [26] investigated the buckling and free vibration of MEE nanoplate resting on Pasternak foundation.

There are also some studies about linear response of MEE shells and curved panels. Bhangale and Ganesan [27] and Annigeri et al. [28] studied free vibration of simply-supported and clamped MEE cylindrical shells, respectively. Wang and Ding [29] developed a method to solve the axisymmetric plane strain dynamic problem of a MEE hollow cylinder subjected to dynamic loads. Tsai et al. [30] solved static problem of doubly-curved functionally graded MEE shells by an asymptotic approach. Tsai and Wu [31 and 32] presented free vibration analysis of simply supported, doubly curved functionally graded MEE shells with open-circuit and closed-circuit

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surface conditions, respectively. Three-dimensional theory has also been used to obtain the free vibration response of MEE cylindrical panels [33]. Lang and Xuewu [34] analyzed buckling and vibration of functionally graded MEE circular cylindrical shells based on the higher order shear deformation theory. Recently, free vibration of a single-layered multiphase MEE shell resting on a Pasternak foundation was studied [35] in which the Donnell shell theory was used to model the plate. Ke et al. [36] developed a MEE cylindrical nanoshell model based on the nonlocal Love's shell theory.

However, there are few studies on the nonlinear behavior of MEE structures. Gao and Zhang [37] and Yu et al. [38] studied nonlinear response of MEE structures. In these studies, the nonlinearity was due to the constitutive equations of MEE material rather than the large deflection motion of the structure. Xue et al. [39] studied the large deflection of a rectangular MEE plate for the first time based on the classical plate theory and Bubnov–Galerkin method. They obtained an analytical relation for the transverse deflection in terms of the applied static force. Later, Sladek et al. [40] used a meshless local Petrov-Galerkin method to study the large deflection of MEE thick plates. Milazzo [41] derived a shear deformable model for the large deflection analysis of MEE laminated plates. Alaimo et al. [42] presented an original finite element formulation based on the first order shear deformation theory (FSDT) for the analysis of large deflections of MEE laminated plates. References [39–42] deals only with the static response of MEE plates and then do not provide any information about the nonlinear oscillation of these smart structures. Recently, Shooshtari and Razavi [43] studied nonlinear free and forced vibration of one-layered multiphase MEE plates based on the classical plate theory. They [44] also investigated nonlinear free vibration of laminated composite plates with MEE layers based on FSDT.

Although there are lots of studies about vibration analysis of doubly-curved shells [45–50], to the best of the authors' knowledge, the analytical study of nonlinear vibration of MEE doubly-curved shells cannot be found in the literature. So, this study is done to fill this gap in analyzing of MEE shells.

The purpose of this paper is to study the linear and nonlinear free vibration of multilayered MEE doubly-curved shells with simply supported boundary condition. To this end, the Donnell shell theory in the presence of rotary inertia effect is used to obtain the equations of motion. Then, Maxwell equations for electrostatics and magnetostatics are used to model the electric and magnetic behavior. Then, the obtained nonlinear PDEs of motion are reduced to a single nonlinear ordinary differential equation (ODE). The resulting equation is solved analytically by Lindstedt-Poincare perturbation method. After validation of the present study, several numerical studies are done to investigate the effects of several parameters on the linear and nonlinear behavior of these smart shells.

2. Modeling of the problem

The Donnell shell theory is used to obtain the equations of motion and the magneto-electro-elastic coupling is introduced to the model through the constitutive equations.

2.1. The Donnell shell theory

Based on the Donnell shell theory, the displacement field of a shallow doubly-curved shell is

$$u = u_0 + z\theta_x, \quad v = v_0 + z\theta_y, \quad w = w_0, \tag{1}$$

where u_0, v_0, and w_0 are the displacements of the mid-surface along x, y, and z directions, respectively, and θ_x and θ_y are the rotations of a transverse normal about the y and x directions, respectively (Fig. 1).

Using the above displacement field, the strain-displacement relations are given as [51]:

$$\begin{aligned}
\varepsilon_x &= \varepsilon_x^0 + z\varepsilon_x^1 = \left(u_{0,x} + w_0/R_x + \frac{1}{2}w_{0,x}^2\right) + z\theta_{x,x},\\
\varepsilon_y &= \varepsilon_y^0 + z\varepsilon_y^1 = \left(v_{0,y} + w_0/R_y + \frac{1}{2}w_{0,y}^2\right) + z\theta_{y,y},\\
\gamma_{xy} &= \gamma_{xy}^0 + z\gamma_{xy}^1 = \left(u_{0,y} + v_{0,x} + w_{0,x}w_{0,y}\right) + z\left(\theta_{x,y} + \theta_{y,x}\right),\\
\gamma_{yz} &= w_{0,y} + \theta_y, \quad \gamma_{xz} = w_{0,x} + \theta_x.
\end{aligned} \tag{2}$$

2.2. Constitutive relations

For the k^{th} layer of a MEE material, the constitutive relations can be written as [9]:

$$\begin{aligned}
\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_{xz}\\ \sigma_{yz}\\ \sigma_{xy}\end{Bmatrix}^{(k)} &= \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0\\ C_{12} & C_{22} & 0 & 0 & 0\\ 0 & 0 & C_{44} & 0 & 0\\ 0 & 0 & 0 & C_{55} & 0\\ 0 & 0 & 0 & 0 & C_{66}\end{bmatrix}^{(k)} \begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xz}\\ \gamma_{yz}\\ \gamma_{xy}\end{Bmatrix}^{(k)}\\
&- \begin{bmatrix} 0 & 0 & e_{31}\\ 0 & 0 & e_{32}\\ 0 & e_{24} & 0\\ e_{15} & 0 & 0\\ 0 & 0 & 0\end{bmatrix}^{(k)} \begin{Bmatrix}E_x\\ E_y\\ E_z\end{Bmatrix}^{(k)}\\
&- \begin{bmatrix} 0 & 0 & q_{31}\\ 0 & 0 & q_{32}\\ 0 & q_{24} & 0\\ q_{15} & 0 & 0\\ 0 & 0 & 0\end{bmatrix}^{(k)} \begin{Bmatrix}H_x\\ H_y\\ H_z\end{Bmatrix}^{(k)},
\end{aligned} \tag{3}$$

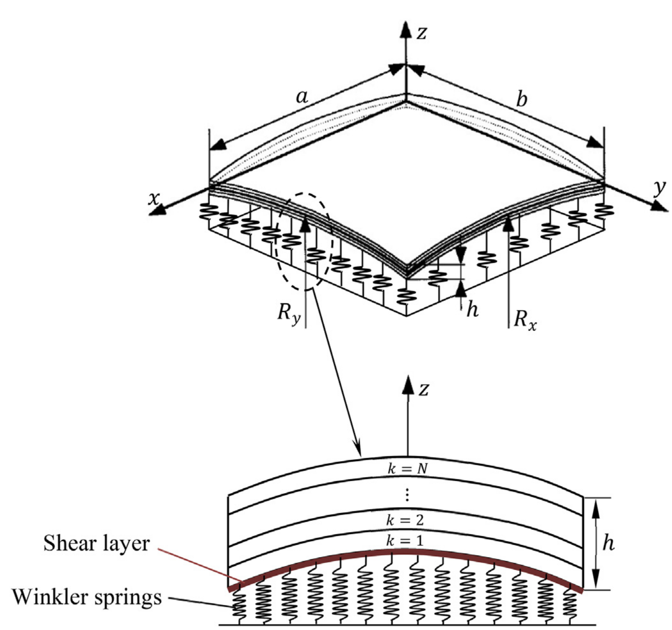

Fig. 1. Schematic of the studied laminated MEE shell.