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Beam transfer functions for relativistic proton bunches with beam–beam interaction

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ABSTRACT

We present a method for the recovery of the transverse tune spread directly from the beam transfer function (BTF). The model is applicable for coasting beams and bunched beams at high energy with a tune spread from transverse nonlinearities induced by the beam–beam effect or by an electron lens. Other sources of tune spread can be added. A method for the recovery of the incoherent tune spread without prior knowledge of the nonlinearity is presented. The approach is based on the analytic model for BTFs of coasting beams, which agrees very well with simulations results for bunched beams at relativistic energies with typically low synchrotron tune. A priori the presented tune spread recovery method is usable only in the absence of coherent modes, but additional simulation data shows its applicability even in the presence of coherent beam–beam modes. Finally agreement of both the analytic and simulation models with measurement data obtained at RHIC is presented. The proposed method successfully recovers the tune spread from analytic, simulated and measured BTF.

Transverse beam transfer functions (BTFs) are a powerful diagnostic tool. They are used for a wide range of applications, the most prominent of which is the detection of the machine tune and the measurement of the stability diagram [1]. Transverse BTFs are measured routinely using baseband Tune (BBQ) systems employing CERNs direct diode detection for increased signal to noise ratio [2]. In coasting beams the BTF can also be used to detect the transverse tune spread from space charge [3] and thereby directly measure the magnitude of the transverse space charge. For bunched beams with high synchrotron frequency, head–tail modes have been observed in BTFs [4].

This paper focuses on the BTF of bunched beams with synchrotron periods of the same order of magnitude as the data acquisition time per BTF sample. We assume that the synchrotron frequency is orders of magnitude below the betatron frequency, conditions commonly found in high energy machines. So far the usual practice is to look at the width of the signal amplitude in either Schottky spectra or BTFs and use it as an estimate for the tune spread. Where the BTF is measured, one also uses the imaginary part of the BTF as an estimate for tune distribution. We present this method and show that it cannot in general be applied to beams with a transverse nonlinearity as a source of the tune spread.

One source of tune spread and the motivation for our study is the electron lens recently installed at the Relativistic Heavy Ion Collider (RHIC) [5]. We want to be able to measure the tune spread it introduces. Other, more widespread sources of tune spread that one might want to quantify using BTFs are space charge or higher order multipoles. An electron lens is a device in which a magnetically confined electron beam of defined shape is guided in parallel to the ion beam in a synchrotron in order to introduce amplitude dependent focusing [6]. Comparable compensation schemes are studied for compensation of space charge [7] at RHIC and another electron lens is discussed for head-on and long-range beam–beam compensation in LHC [8]. At RHIC the electron lens will be used to partly compensate the incoherent tune spread from the beam–beam interaction in proton operation [5]. We want to diagnose its effect on the tune spread using the BTF. To achieve this goal, this paper makes use of an analytic model for the BTF of a local nonlinear lens building on the existing theory for coasting beams by Berg and Ruggiero [9] based on earlier work by Hereward [10]. For the simulation model with Gaussian beams the tune spread and shape can be recovered by means of fitting the BTF against the presented analytic model if the shape of the nonlinearity is known. In order to treat the more general case in which the shape of the nonlinearity is not known, a method to measure the tune spread directly from the BTF is introduced. We refer to it as *threshold method*. The two methods are applied to analytic and simulated BTFs. We proceed to discuss the beam–beam effect as a localized nonlinear lens and compare analytic results and simulations to measurements.

In the first section we discuss the analytic equation for the BTF of coasting beams undergoing an electron–lens or beam–beam

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interaction. These equations are valid in the limit of absence of coherent modes. We present a robust method to determine tune spread for arbitrary beam shapes and nonlinearities via detection of Landau damping.

In the second section we introduce our simulation model for BTFs with the beam–beam effect and electron lenses. We compare its results to the analytic expectations and obtain agreement with the analytic expectation. We show that the tune spread determination method introduced in the analytic section applies also in the presence of coherent beam–beam modes in favorable conditions. We argue that the coasting beam equations stay valid when the synchrotron frequency is much lower than the betatron frequency.

In the last section we compare simulation and analytic results to measurements from a dedicated machine experiment. We find BTFs in agreement with our analytic expectations, for both the full BTF shape and the threshold method introduced in the first section.

1. Analytic model

In this section we introduce an analytic model based on the theory for coasting beams, which is extended to bunched beams in the following sections. We investigate methods to reconstruct the transverse tune distribution of the beam caused by a local nonlinearity in the horizontal and the vertical plane. We motivate our approach by revisiting well-known analytic results for incoherent transverse BTFs of coasting beams with tune spread from chromaticity. We show that in this case the tune distribution can be easily recovered. With this in mind we present a model for the BTF due to a transverse nonlinearity. We show that even for a flat beam, the recovery of the tune distribution is not possible without prior knowledge of the exact shape of the nonlinearity that gives rise to the tune spread. For this reason, we then present a method for recovery of only the total tune spread, not the shape, from the BTF via detection of Landau damping. Because it detects Landau damping, the method works without prior knowledge of the source of the tune spread. Tune spread from chromaticity, octupoles, an electron lens, incoherent space charge or any other source will be detected in the absence of coherent modes and external damping.

1.1. Tune spread from chromaticity

The transverse beam transfer function $R(\Omega)$ is defined as the fraction of the complex response amplitude $A(\Omega)$ of the beam per driving amplitude $D(\Omega)$ of a beam excited at the frequency Ω. In the following we will often write *BTF* for brevity when we in fact refer to the *transverse BTF*:

$$R(\Omega)=\frac{A(\Omega)}{D(\Omega)} \tag{1}$$

It is understood that the BTF is meaningful when taken at small amplitudes where D scales linearly with A. Note that for our considerations we need the complex value of the BTF.

There is a good amount of research on BTF of coasting beams. One well-known example that can be found in textbooks such as [11] is the case of beams with a tune spread originating from momentum spread and chromaticity. Assuming that the density of particles in the beam ψ is known as a function of the betatron frequency ω, the BTF R at frequency Ω can be calculated as

$$R(\Omega)\propto\int_{-\infty}^{\infty}\frac{1}{\omega-\Omega}\psi(\omega)\,\mathrm{d}\omega \tag{2}$$

The pole at $\Omega=\omega$ can be accounted for by adding a small imaginary term to the denominator or by means of the residue theorem. Finally one arrives at

$$R(\Omega)\propto -i\pi\psi(\Omega)+\text{P.V.}\int_{-\infty}^{\infty}\frac{1}{\omega-\Omega}\psi(\omega)\,\mathrm{d}\omega \tag{3}$$

wherein P.V. denotes Cauchys principal value integral. In the case of chromaticity the BTF is described by Eq. (3) and therefore the betatron frequency distribution is proportional to the imaginary part of the BTF. Measuring the imaginary part of the BTF directly gives the betatron frequency distribution in the plane of the BTF.

1.2. Beam transfer functions due to localized transverse forces

When the tune depends on the amplitude of the particle in the plane of the BTF excitation, the situation becomes more complicated. This is the case for nonlinear elements like octupoles or an electron lens. Here the particles' change in amplitude due to excitation leads to a consequent change in its betatron frequency which makes the treatment of the BTF less trivial. We build on work by Berg and Ruggiero for coasting beams. They derived the BTF of a beam with tune spread due to a localized octupole [9]. Their form for the BTF in the i direction R_i (with i either x or y) reads

$$R_i(\Omega)=c\cdot\int_0^{\infty}\int_0^{\infty}\frac{1}{\Omega-\omega_i\left(J_x,J_y\right)}\frac{J_i\mathrm{d}\psi}{\mathrm{d}J_i}\mathrm{d}J_x\,\mathrm{d}J_y \tag{4}$$

wherein c is a constant, J_x,J_y the transverse action angle variables, ψ the distribution function in action angle variables and $\omega_i(J_x,J_y)$ the betatron frequency as a function of these variables. Ω is the frequency at which the BTF is calculated. Note that this equation is still general and becomes octupole specific later in [9] by the introduction of the $\omega_i(J_x,J_y)$ for an octupole. An equation for tune shift caused by a circular Gaussian charge distribution for space charge in these coordinates was given by Burov and Lebedev [12]. We use it as follows:

$$\omega_x(J_x,J_y)=\omega_{0,x}+\xi_{\mathrm{bb}}\int_0^1\frac{\left(I_0\left(\frac{J_xz}{2}\right)-I_1\left(\frac{J_xz}{2}\right)\right)I_0\left(\frac{J_yz}{2}\right)}{\exp\left(z(J_x+J_y)/2\right)}\mathrm{d}z \tag{5}$$

with ξ_{bb} being the maximum tune shift (for particles in the center of the beam), I_0 and I_1 the modified Bessel functions and $\omega_{0,x}$ the lattice tune in the x direction. ω_y can be found by exchanging x and y in the righthand side of Eq. (5). The equation is equivalent to the one given in [13] for beam–beam evaluated for round beams. We use it as a model for a stationary electron lens which, in RHIC, is also a Gaussian charge distribution. For ψ we use a Gaussian beam [9]: $\psi(J_x,J_y)=\sigma^{-4}\exp(\sigma^{-2})\exp(-J_x-J_y)$ unless noted otherwise.

If needed, chromaticity and other sources of tune spread can be added to Eq. (4) by inserting its contribution to ω_i and ψ and adding an integration over momentum if necessary.

1.3. Flat beam case

Compared to the case of tune spread due to chromaticity, the recovery of the tune spread from transverse sources is much harder, mostly due to the two-dimensional nature of the problem: the particle tune is a function of both J_x and J_y and there is no way for the BTF to directly determine J_x. Generally the equitune lines will not be parallel to the coordinate axes in J_x,J_y, making it impossible to associate one J_x to one tune shift in the full 2D case. To make the problem more tractable one may first assume a very flat distribution. This would for example be the case for a beam of very low vertical emittance. Say

$$\psi(J_x,J_y)=\begin{cases}\varepsilon^{-1}\psi_x(J_x), & J_y<\varepsilon\\ 0, & J_y\geq\varepsilon\end{cases} \tag{6}$$