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X-ray differential phase-contrast tomographic reconstruction with a phase line integral retrieval filter

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A B S T R A C T

We report an alternative reconstruction technique for x-ray differential phase-contrast computed tomography (DPC-CT). This approach is based on a new phase line integral projection retrieval filter, which is rooted in the derivative property of the Fourier transform and counteracts the differential nature of the DPC-CT projections. It first retrieves the phase line integral from the DPC-CT projections. Then the standard filtered back-projection (FBP) algorithms popular in x-ray absorption-contrast CT are directly applied to the retrieved phase line integrals to reconstruct the DPC-CT images. Compared with the conventional DPC-CT reconstruction algorithms, the proposed method removes the Hilbert imaginary filter and allows for the direct use of absorption-contrast FBP algorithms. Consequently, FBP-oriented image processing techniques and reconstruction acceleration softwares that have already been successfully used in absorption-contrast CT can be directly adopted to improve the DPC-CT image quality and speed up the reconstruction.

1. Introduction

X-ray phase-contrast computed tomography (PC-CT) uses the phase shift that x-rays undergo when passing through matter, rather than their attenuation, as the imaging signal may provide better image quality in soft tissue and low atomic number samples. Over the last years, several PC-CT methods have been developed [1–22]. One of the recent developments is differential PC-CT (DPC-CT), based on a grating interferometer [14–22]. It includes essentially two steps: (i) retrieval of the DPC projections and (ii) phase reconstruction. The first step can be accomplished by using a phase-stepping procedure [15–17,19], a reverse projection method [23], or a single-shot Fourier-based phase-extraction method [24]. The second step has so far been solved by using an analytic algorithm with a Hilbert filter [20,25,26], or algebraic iterative reconstruction techniques [27], or back-projection filter (BPF) algorithms [28].

In the field of absorption-based x-ray CT, filtered back-projection (FBP) algorithms [29] have been popular because of their high calculation efficiency and reconstruction accuracy. Many FBP-oriented image processing techniques have been developed to improve imaging quality, including spatial resolution enhancement [30], beam-hardening correction [31], metal artifact deletion [32] and ring artifact correction [33]. Specific commercial reconstruction acceleration softwares [34,35] have also been developed to improve the speed of FBP algorithms. Traditional FBP algorithms use the assumption that the image recorded by the x-ray detector is the line integral projection of the reconstructed object function. In the case of DPC-CT, however, the experimental arrangement yields the line integral projections of the partial derivative of the object function. This prevents DPC-CT from adopting existing FBP-oriented image processing techniques and reconstruction acceleration software. Although the direct integration of the DPC data is quite straightforward, the exact value of the first differential data should be known exactly. Unfortunately in practice, it is very difficult to do so due to the noise and the error from the data acquisition.

Here we report an alternative reconstruction technique with a new phase line integral retrieval filter and its experimental results, which demonstrate the feasibility of DPC-CT reconstruction with standard, absorption-contrast FBP algorithms. This approach will be of particular interest for future medical and industrial applications of x-ray DPC-CT, because it allows DPC-CT to directly adopt the existing absorption-contrast CT image processing techniques and the reconstruction acceleration software, that have already been successfully used in clinical CT, to improve the image quality and the speed.

2. Methods and materials

In the following sections, we consider a two-dimensional object as shown in Fig. 1. It can be described by a complex refractive index distribution $n(x,y) = 1 - \delta(x,y) + i\beta(x,y)$, where x

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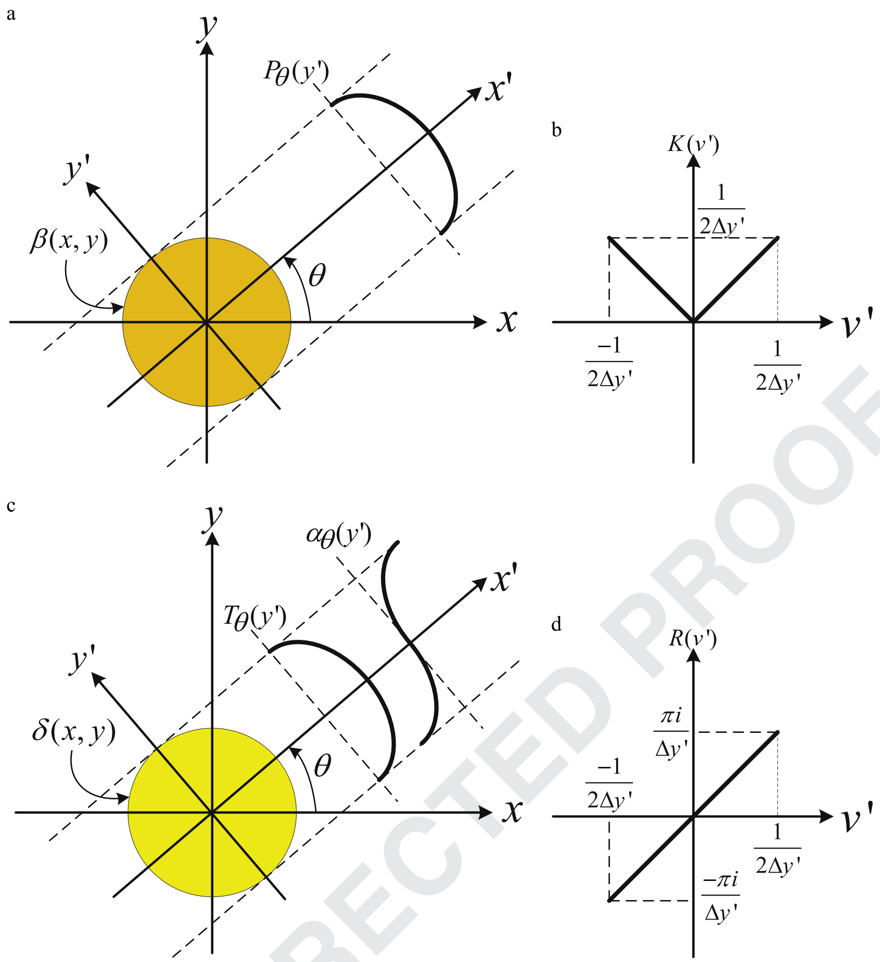

Fig. 1. Tomographic reconstruction. (a) and (c) are projection geometries for absorption-contrast and differential phase-contrast CT. (b) is the filter for line integral projection. (d) is the phase line integral retrieval filter. $\Delta y'$ is the effective detector pixel size.

and y describe the coordinate system of the sample. In absorption-contrast CT, β is measured by the attenuation of the x-rays transmitted through the specimen. The transmission projection through the object can be described by the Radon transform of the object [29]:

$$P_\theta(y') = \int_{-\infty}^{\infty} \beta(x', y')\, dx' \tag{1}$$

where x' and y' denote a coordinate system which is rotated by an angle θ with respect to x and y. To reconstruct β from a set of projection images $P_\theta(y')$ described by Eq. (1), the FBP algorithm is usually employed and can be written as [29]

$$\beta(x, y) = \int_0^{\pi} FT^{-1}[\overline{P}_\theta(v')K(v')]\, d\theta \tag{2}$$

where $\overline{P}_\theta(v')$ represents the Fourier transform of the transmission projection $P_\theta(y')$, v' is the Fourier space coordinate corresponding to the real space coordinate y' and FT^{-1} denotes the inverse Fourier transform operator. $K(v')$ is the Ram–Lak filter in the Fourier space and given by $K(v') = |v'|$, depicted in Fig. 1(b).

In differential phase-contrast imaging, one measures the effect of variations of the refraction index decrement δ by evaluating the tiny refraction angles of x-rays induced by the specimen with a grating Talbot interferometer. Under the paraxial approximation, the refraction angle can be expressed by [20]

$$\alpha_\theta(y') = \int_{-\infty}^{\infty} \frac{\partial \delta(x', y')}{\partial y'}\, dx'. \tag{3}$$

Comparing Eq. (3) with Eq. (1), we know that the measured DPC data is not the line integral projection of δ but its partial derivative. Then reconstruction based on FBP in Eq. (2) will not result in a correct δ. For an accurate reconstruction in DPC-CT, the analytical algorithm based on the Hilbert imaginary filter was proposed and expressed by [20,25]

$$\delta(x, y) = \int_0^{\pi} FT^{-1}[\overline{\alpha}_\theta(v')H(v')]\, d\theta \tag{4}$$

where $\overline{\alpha}_\theta(v')$ represents the Fourier transform of the DPC projection $\alpha_\theta(y')$ and $H(v')$ is the Hilbert filter given by $H(v') = (1/2\pi)i\mathrm{sgn}(v')$.

According to Theorem A13 in [36], if δ and $\partial\delta/\partial y'$ are continuous in a finite support $[a\ b]$ (satisfied in DPC-CT after the phase-unwrapping correction), Eq. (3) may be also expressed by

$$\alpha_\theta(y') = \int_b^a \frac{\partial \delta(x', y')}{\partial y'}\, dx' = \frac{\partial \int_b^a \delta(x', y')\, dx'}{\partial y'}. \tag{5}$$

The derivative property of the Fourier transform states that taking the Fourier transform of a derivative is the same as taking the Fourier transform of the original function and multiplying by $2\pi i v'$. Using this property and Eq. (5), we have

$$\overline{\alpha}_\theta(v') = FT(\partial T_\theta(y')/\partial y') = 2\pi i v' \overline{T}_\theta(v') \tag{6}$$

with

$$T_\theta(y') = \int_b^a \delta(x', y')\, dx' \tag{7}$$